

Example 4. Orange juice in a cylindrical container, 0.30 m diameter by 0.45 m tall, is to be frozen in a blast freezer. The initial temperature of the juice is 5°C and the freezer air temperature is −35°C. The surface heat transfer coefficient is estimated to be 30 W/(m²·K). Calculate the time required for the thermal center of the juice to reach −18°C.

Solution: Because the food is a finite cylinder, the algorithm based on the method of equivalent heat transfer dimensionality (Cleland et al. 1987a, 1987b) is used. This method requires calculation of the freezing time of an infinite slab, which is determined using the method of Hung and Thompson (1983).

Step 1: Determine the thermal properties of orange juice.

Using the methods described in [Chapter 9](#), the thermal properties of orange juice are calculated as follows:

Property	At −40°C (Fully Frozen)	At −18°C (Final Temp.)	At 5°C (Initial Temp.)
Density, kg/m ³	$\rho_s = 970$	$\rho_s = 970$	$\rho_l = 1038$
Enthalpy, kJ/kg	—	$H_s = 40.8$	$H_l = 381.5$
Specific Heat, kJ/(kg·K)	$c_s = 1.76$	—	$c_l = 3.89$
Thermal Cond., W/(m·K)	$k_s = 2.19$	—	—
Initial freezing temperature: $T_f = -0.4^\circ\text{C}$			

Volumetric enthalpy difference between $T_i = 5^\circ\text{C}$, and -18°C :

$$\Delta H_{18} = \rho_l H_l - \rho_s H_s$$

$$\Delta H_{18} = (1038)(381.5) - (970)(40.8) = 356 \times 10^3 \text{ kJ/m}^3$$

Volumetric specific heats:

$$C_s = \rho_s c_s = (970)(1.76) = 1707 \text{ kJ/(m}^3\cdot\text{K)}$$

$$C_l = \rho_l c_l = (1038)(3.89) = 4038 \text{ kJ/(m}^3\cdot\text{K)}$$

Step 2: Determine the surface heat transfer coefficient.

The surface heat transfer coefficient is estimated to be 30 W/(m²·K).

Step 3: Determine the characteristic dimension D and the dimensional ratios β_1 and β_2 .

For freezing time problems, the characteristic dimension is twice the shortest distance from the thermal center of the food item to its surface. For the cylindrical sample of orange juice, the characteristic dimension is equal to the diameter of the cylinder:

$$D = 0.30 \text{ m}$$

Using Equations (16) and (17), the dimensional ratios then become

$$\beta_1 = \beta_2 = \frac{0.45 \text{ m}}{0.30 \text{ m}} = 1.5$$

Step 4: Using Equations (28) to (30), calculate the Biot, Plank, and Stefan numbers.

$$\text{Bi} = \frac{hD}{k_s} = \frac{(30.0)(0.30)}{2.19} = 4.11$$

$$\text{Pk} = \frac{C_l(T_i - T_f)}{\Delta H_{18}} = \frac{(4038)[5 - (-0.4)]}{356 \times 10^3} = 0.0613$$

$$\text{Ste} = \frac{C_s(T_f - T_m)}{\Delta H_{18}} = \frac{(1707)[-0.4 - (-35)]}{356 \times 10^3} = 0.166$$

Step 5: Calculate the freezing time of an infinite slab.

Use the method of Hung and Thompson (1983). First, find the weighted average temperature difference given by Equation (33).

$$\begin{aligned} \Delta T &= [-0.4 - (-35)] \\ &+ \frac{[5 - (-0.4)]^2(4038/2) - [-0.4 - (-18)]^2(1707/2)}{356 \times 10^3} = 34.0 \text{ K} \end{aligned}$$

Determine the parameter U :

$$U = \frac{34.0}{-0.4 - (-35)} = 0.983$$

Determine the geometric parameters P and R for an infinite slab using Equations (34) and (35):

$$\begin{aligned} P &= 0.7306 - (1.083)(0.0613) \\ &+ (0.166) \left[(15.40)(0.983) - 15.43 + \frac{(0.01329)(0.166)}{4.11} \right] = 0.616 \end{aligned}$$

$$R = 0.2079 - (0.2656)(0.983)(0.166) = 0.165$$

Determine the freezing time of the slab using Equation (36):

$$\theta = \frac{3.56 \times 10^8}{34.0} \left[\frac{(0.616)(0.30)}{30.0} + \frac{(0.165)(0.30)^2}{2.19} \right] = 135\,000 \text{ s} = 37.5 \text{ h}$$

Step 6: Calculate the equivalent heat transfer dimensionality for a finite cylinder.

Use the method presented by Cleland et al. (1987a, 1987b), Equations (60) to (63), to calculate the equivalent heat transfer dimensionality. From [Table 7](#), the geometric constants for a cylinder are

$$G_1 = 2 \quad G_2 = 0 \quad G_3 = 1$$

Calculate E_2 :

$$\phi = \frac{2.32}{\beta_2^{1.77}} = \frac{2.32}{1.5^{1.77}} = 1.132$$

$$X(1.132) = \frac{1.132}{4.11^{1.34} + 1.132} = 0.146$$

$$E_2 = \frac{0.146}{1.5} + (1 - 0.146) \frac{0.50}{1.5^{3.69}} = 0.193$$

Thus, the equivalent heat transfer dimensionality E becomes

$$E = G_1 + G_2 E_1 + G_3 E_2$$

$$E = 2 + (0)(E_1) + (1)(0.193) = 2.193$$

Step 7: Calculate freezing time of the orange juice using Equation (55):

$$\theta_{shape} = \theta_{slab} / E = 135\,000 / 2.193 = 61\,600 \text{ s} = 17.1 \text{ h}$$