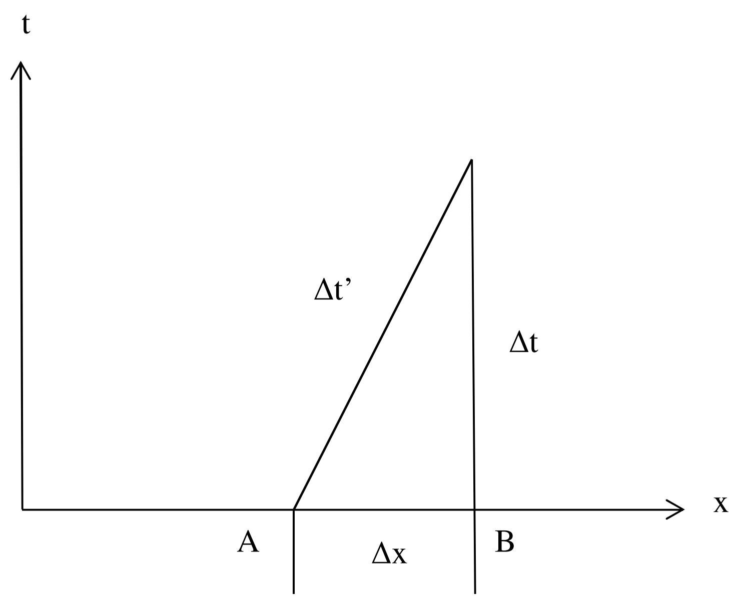
Einstein's clock example:

- Clocks A and B are synchronized in a stationary reference frame.
- Clock A is moved to the location of clock B.
- Clock A has lost some time.

Worldlines:



Lorentz Transform:

$$\begin{Bmatrix} \Delta x'(=0) \\ c\Delta t' \end{Bmatrix}_A = \gamma \begin{bmatrix} 1 & -\beta \\ -\beta & 1 \end{bmatrix} \begin{Bmatrix} \Delta x \\ c\Delta t \end{Bmatrix}_B$$

Solve by inverting the matrix:

$$\begin{Bmatrix} \Delta x \\ c\Delta t \end{Bmatrix}_B = \gamma \begin{bmatrix} 1 & +\beta \\ +\beta & 1 \end{bmatrix} \begin{Bmatrix} \Delta x'(=0) \\ c\Delta t' \end{Bmatrix}_A$$

Thus: $\Delta t = \gamma \Delta t'$ Since: $1 \le \gamma$ Then: $\Delta t' \le \Delta t$

Clock A has lost some time. Or with the Minkowski metric:

$$
\begin{aligned}
(c\Delta t')^2 &= (c\Delta t)^2 - (\Delta x)^2 \\
\left(\frac{c\Delta t'}{c\Delta t}\right)^2 &= 1 - \left(\frac{\Delta x}{c\Delta t}\right)^2 \\
&= 1 - \beta^2 \\
&= 1/\gamma^2
\end{aligned}
$$

So again: $\Delta t = \gamma \Delta t'$

And: $\Delta t' \le \Delta t$