1.5-2 Consider the incompressible Newtonian fluid flowing, under the pressure gradient dp/dz, between two stationary horizontal plates separated by a gap of $2L$, as shown in Fig. P1.5-2 (see p. 108). The volume flow rate Q is known. Find the steady-state velocity distribution and the shear force acting on the two plates by the fluid.

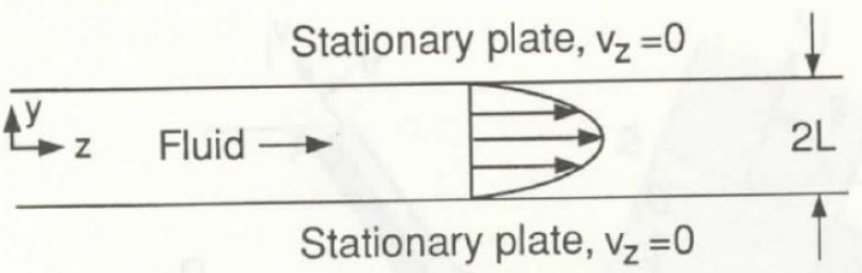

Fig. P1.5-2

Equation of continuity in rect. coord.:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\rho v_x + \frac{\partial}{\partial y}\rho v_y + \frac{\partial}{\partial z}\rho v_z = 0$$

After simplification:

$$\frac{\partial}{\partial y} v_y = 0$$

Now the equation of motion in rect coord. in the z direction:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial y} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right)$$

After simplification:

$$\mu\frac{\partial^2 v_z}{\partial y^2} - \frac{dP}{dz} = 0$$