Parallel Transport through Inertial Frames

We have the Schwarzschild metric for m=1/2

$$ds^2 = (1-\frac{1}{r})dt^2 - (1-\frac{1}{r})^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

In the above metric we hold r constant making r=R

We keep time variable . In the final stage we will consider $dt \to 0$ so that time becomes constant and we are back in a loop.

We have for the 45^0 latitude line:

$$ds^2 = (1-\frac{1}{R})dt^2 - R^2 d\theta^2 - R^2\frac{1}{2}d\varphi^2$$

We use the transformations:

$$T = \sqrt{(1-\frac{1}{R})}t$$

$$x = R\theta$$

$$y = \frac{1}{\sqrt{2}}\varphi R$$

The last two transformations apply locally.

Regarding time: It represents physical time [having the dimension of time]

Regarding x: The x-axis is parallel to the e^θ vector

Regarding y: The y-axis is parallel to the e^φ vector

[important to note that we have a z-axis along e^r]

Our metric now:

$$ds^2 = dT^2 - dx^2 - dy^2$$

Movement along the latitude:

We consider an infinitesimal movement of the coordinate frame along the line of latitude.We come to the new position of the $(e^\varphi, e^\theta, e^r)$ triad ,corresponding to the (x,y,z) triad The There are two effects:

1) An infinitesimal translation that leaves the direction of the axes unchanged.

$x = x' + h$

$y = y' + k$

$z = z' + l$

The metric now:

$ds^2 = dT^2 - dx'^2 - dy'^2 - dz'^2$

2) A infinitesimal rotation which may be expressed by a matrix $A = a_{ij}$ consisting of the Eulerian angles.

We leave the translation unchanged but we apply the inverse transformation A^{-1} to cancel the effect of rotation.

Form of the metric:

$ds^2 = dT^2 - dx'^2 - dy'^2 - dz'^2$

We carry on this process as we move along the line of latitude
It is to be noted that the axes do not change their orientation as we move from one frame to another. The frames are not in relative motion.

Parallel Transport Equation in each frame:

$$\frac{dA^{\mu}}{dx^{\nu}} = 0$$

The components do not change as we pass in the same frame. They do not change as we pass from one frame to another.
The vector does not change its orientation at any instant.

Movement along the meridian: Similar arguments may be applied in the case of a meridian. We simply move the [(x,y,z)->$(e^{\varphi}, e^{\theta}, e^{r})$]triad along the meridian instead of the latitude. Each small movement can be decomposed in to a translation and a rotation. We reverse the effect of rotation keeping the translation in tact.
Net Effect:

1) The metric has the diagonal form[1-1-1-1]
2) In each situation the axes remain parallel to the previous situation[respectively]
3) There is no relative motion between the frames.
4) In each frame we have $\frac{dA^{\mu}}{dx^{\nu}} = 0$
5) As we pass from frame to frame the components remain unchanged