Basic Trigonometric Identities

Let x and y be any real numbers (restricted so that both sides of an equation are defined). Hence, x and y are in radian measure.

Reciprocal Identities

1. $\csc x = \dfrac{1}{\sin x}$ 2. $\sec x = \dfrac{1}{\cos x}$ 3. $\cot x = \dfrac{1}{\tan x}$

Quotient Identities

4. $\tan x = \dfrac{\sin x}{\cos x}$ 5. $\cot x = \dfrac{\cos x}{\sin x}$

Identities for Negatives, i.e. defines function as Even or Odd. The other three are straight-forward computations using the definitions of Even and Odd Functions.

6. $\sin(-x) = -\sin x$ 7. $\cos(-x) = \cos x$ 8. $\tan(-x) = -\tan x$

Pythagorean Identity

9. $\sin^2 x + \cos^2 x = 1$ 10. $1 + \tan^2 x = \sec^2 x$ 11. $1 + \cot^2 x = \csc^2 x$

Sum Identities

12. $\sin(x + y) = \sin x \cos y + \cos x \sin y$
13. $\cos(x + y) = \cos x \cos y - \sin x \sin y$
14. $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

Difference Identities

15. $\sin(x - y) = \sin x \cos y - \cos x \sin y$
16. $\cos(x - y) = \cos x \cos y + \sin x \sin y$
17. $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

CoFunction Identities

18. $\cos\left(\dfrac{\pi}{2} - y\right) = \sin y$ 19. $\sin\left(\dfrac{\pi}{2} - y\right) = \cos y$ 20. $\tan\left(\dfrac{\pi}{2} - y\right) = \cot y$

Double-Angle Identities

21. $\sin 2x = 2 \sin x \cos x$
22. $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
23. $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x} = \dfrac{2 \cot x}{\cot^2 x - 1} = \dfrac{2}{\cot x - \tan x}$

Half-Angle Identities

24. $\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

25. $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

26. $\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos} = \frac{1-\cos x}{\sin x}$

Note: the sign is determined by the quadrant in which $\frac{x}{2}$ lies.

Product-Sum Identities

27. $\sin x\cos y = \frac{1}{2}[\sin(x+y)+\sin(x-y)]$
28. $\cos x\sin y = \frac{1}{2}[\sin(x+y)-\sin(x-y)]$
29. $\sin x\sin y = \frac{1}{2}[\cos(x-y)-\cos(x+y)]$
30. $\cos x\cos y = \frac{1}{2}[\cos(x+y)+\cos(x-y)]$

Sum-Product identities

31. $\sin x+\sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$
32. $\sin x-\sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$
33. $\cos x+\cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$
34. $\cos x-\cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

Solving Triangles.
Law of Sines and Cosines

For any triangle with vertices A, B, C whose angles are α, β, γ and opposite sides are a, b, c respectively.

35. $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$
36. $a^2 = b^2 + c^2 - 2bc\cos\alpha$
37. $b^2 = a^2 + c^2 - 2ac\cos\beta$
38. $c^2 = a^2 + b^2 - 2ab\cos\gamma$