6an 19. $B = \begin{pmatrix} 2 & -3 & -1 \\ 2 & -3 & -1 \\ -3 & 3 & 2 \end{pmatrix}$

To find CS(B), we need to find the ~~REF(B) in order to~~ determine the basis for ~~CS(A)~~ CS(B). This can be done by testing if each column vector is linearly independent

linear independence

$$\lambda_1 \underline{v_1} + \lambda_2 \underline{v_2} + \dots + \lambda_n \underline{v_n} = 0$$

$\lambda_1 = \lambda_2 = \dots = \lambda_n = 0$ to verify linear independence

~~Trying~~

$$a(2,2,-3)^T + b(-3,3,3)^T + c(-1,-1,2)^T = (0,0,0)^T \quad D$$

$2a - 3b - c = 0$ ① $\Rightarrow c = 2a - 3b$

$-3a + 3b + 2c = 0$ ②

$-3a + 3b + 4a - 6b = 0$

$a - 3b = 0 \Rightarrow a = 3b$

substitute $a = 3b$ into ①

$2a - a - c = 0 \therefore a = c$, $b = \frac{a}{3} = \frac{c}{3}$

Linear independence says that the coefficients must all be 0 to satisfy

~~as~~ Since $a = c$ D becomes

$$a(2,2,-3)^T + a(-1,-1,2)^T + b(-3,-3,3)^T = a(1,1,-1)^T + b(-3,-3,3)^T$$