← ☆ indicates correction to original paper.

The observable universe inside a black hole

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A Schwarzschild radial coordinate R is presented for the Friedmann dust-filled cosmology models. It is shown that a worldline of constant Schwarzschild radial coordinate in the dust-filled universe is instantaneously null at $R_n=2GM/c^2$, where M is the Schwarzschild mass inside the sphere $R=R_n$. It is also shown that $M_p=3\tau c^3/4G$, where M_p is the proper mass inside $R=R_n$ and τ is the age of the universe. The $R_n=2GM/c^2$ result in Friedmann dust-filled cosmology is made physically significant by abandoning the cosmological principle and adjoining segments of Friedmann dust to segments of Schwarzschild vacuum. In the resulting cosmology model, the observable universe may lie inside a black or white hole.

I. INTRODUCTION

It is not uncommon to find introductory astronomy texts or popularized literature stating that the Milky Way and all the galaxies of the observable universe may lie inside a black hole. However, these sources rarely establish the general relativistic (GR) context for the "black hole universe."[1] In this paper, we explain a GR cosmology model in which the observable universe may lie inside a black or white hole.

We assume the reader is familiar with GR cosmology and the Schwarzschild solution with its Kruskal extension. Those not familiar with these topics are referred to a few of many excellent introductions.[2]

We begin in Sec. II by introducing Schwarzschild coordinates in GR cosmology.[3] We show that the worldline of constant Schwarzschild radial coordinate in the dust-filled universe is instantaneously null at $R_n=2GM/c^2$, where M is the Schwarzschild mass inside the sphere $R=R_n$. The Hubble sphere and various horizons are briefly discussed. We then show the proper mass $M_p=3\tau c^3/4G$ inside the sphere $R=R_n$ for the dust-filled universe, where τ is the age of the universe.

The result $R_n=2GM/c^2$ of Sec. II is only suggestive. We actualize this implication in Sec. III, where we discard the cosmological principle and construct a cosmology model from a combined Friedmann dust and Schwarzschild vacuum. The merger is outlined for each of the dust-filled models. The results are discussed in Sec. IV, emphasizing the conditions whereby the observable universe exists inside a black or white hole.

While this result is not necessarily of astrophysical importance, the methodology has been used extensively by cosmologists and astrophysicists. The methods outlined in this paper were used as early as 1939 by Oppenheimer and Snyder[4] to explain stellar collapse. Examples of more recent uses of this technique: Farhi and Guth[5] examined the possibility of creating an inflationary universe in the laboratory; Frolov, Markov, and Mukhanov[6] studied spacetime inside of a black hole; Goldwirth[7] found a basic asymmetry between the expanding and collapsing phases of the closed universe; Harwit[8] offered an explanation of the large scale structure of the universe; and Dyer, Landry, and Shaver[9] joined the flat dust-filled Friedmann model with one of the spatially homogeneous, anisotropic, vacuum spacetimes of the Kasner metric.

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II. SCHWARZSCHILD COORDINATES IN COSMOLOGY

Before we outline the merger of Friedmann dust-filled cosmologies with Schwarzschild space, we hint at the results of such a combination. We do this by defining a radial coordinate R in the dust-filled Friedmann models analogous to the radial coordinate of Schwarzschild space. Using the form

$$ds^2=-c^2d\tau^2+a^2(\tau)\left(d\chi^2+\begin{matrix}\sin^2\chi\,d\Omega^2\\ \chi^2\,d\Omega^2\\ \sinh^2\chi\,d\Omega^2\end{matrix}\right) \tag{1}$$

for the closed, flat, and open Robertson-Walker metric respectively, we have

$$\begin{aligned}R&=a(\tau)\sin\chi\\ &=a(\tau)\chi\\ &=a(\tau)\sinh\chi\end{aligned} \tag{2}$$

in each of these models. Because the universe is expanding, an observer trying to maintain constant R would have a local velocity with respect to the comoving observers. At large enough R (call it R_n), this velocity would be c, i.e., the worldline would be null. This is analogous to $r=2GM/c^2$ in the Schwarzschild metric.

Also analogous to the Schwarzschild space, $R_n=2GM/c^2$ where M is the Schwarzschild mass inside $R=R_n$, that is[10]

$$M=4\pi\int_0^{R_n}\rho(R)R^2\,dR. \tag{3}$$

In the Friedmann dust-filled cosmologies, ρ is constant on a constant τ surface, so we have simply

$$M=\frac{4\pi R_n^3\rho}{3} \tag{4}$$

for all models. That the null observer be at constant R $(=R_n)$ gives

$$\begin{aligned}\frac{dR}{d\tau}&=a\cos\chi\frac{d\chi}{d\tau}+\sin\chi\frac{da}{d\tau}=0\\ &=a\frac{d\chi}{d\tau}+\chi\frac{da}{d\tau}=0\\ &=a\cosh\chi\frac{d\chi}{d\tau}+\sinh\chi\frac{da}{d\tau}=0\end{aligned} \tag{5}$$

in the closed, flat, and open models. Since $a(d\chi/d\tau)=c$ is the local velocity of the null observer

$$\begin{aligned}c&=\tan\chi_n\frac{da}{d\tau}\\ &=\chi_n\frac{da}{d\tau}\\ &=\tanh\chi_n\frac{da}{d\tau}\end{aligned} \tag{6}$$

(ignoring algebraic sign) where χ_n is the comoving radial coordinate at R_n. Using the Friedmann dust-filled solutions, we have

$$\begin{aligned}\frac{da}{d\tau}&=\frac{c}{\tan\eta/2}\\ &=\left(\frac{9B}{4}\right)^{1/3}\frac{2}{3\tau^{1/3}}\,c^{2/3}\\ &=\frac{c}{\tanh\eta/2}\end{aligned} \tag{7}$$

where

$$B=a\dot{a}^2=\frac{8\pi G\rho a^3}{3c} \tag{8}$$

(is a constant) and

$$c\,d\tau=a\,d\eta \tag{9}$$

defines η.
Equations (6) and (7) give

$$\begin{aligned}\chi_n&=\eta/2\\ &=\frac{3\cancel{}\tau^{1/3}}{2}\left(\frac{4}{9B}\right)^{1/3}c^{1/3}\\ &=\eta/2.\end{aligned} \tag{10}$$

Using the dust-filled solutions with Eqs. (4), (8), and (10) yields

$$\begin{aligned}\frac{2GM}{c^2}&=\frac{B\sin^2(\eta/2)\sin\chi_n}{\cancel{}}=a\sin\chi_n=R_n\\ &=\frac{3c\tau}{2}=a\chi_n=R_n\\ &=\frac{B\sinh^2(\eta/2)\sinh\chi_n}{\cancel{}}=a\sinh\chi_n=R_n\end{aligned} \tag{11}$$

as claimed.

Thus upon defining a Schwarzschild radial coordinate R in Friedmann dust-filled cosmology, we find the following analogy with Schwarzschild space. There is a sphere $R=R_n$ about the origin on which constant R worldlines are null, and the Schwarzschild mass M within this sphere is $R_nc^2/2G$.

Non-Schwarzschild-like characteristics should also be noted. Equation (11) does not hold between the proper mass[11] M_p and R_n, except in the flat model where space is Euclidean. Rather, in all three models $M_p=3\tau c^3/4G$ inside $R=R_n$. To see this, first compute

$$\begin{aligned}M_p&=4\pi\rho a^3\int_0^{\chi_n}\sin^2\chi\,d\chi=4\pi\rho a^3\left(\frac{\chi_n}{2}-\frac{\sin 2\chi_n}{4}\right)\\ &=4\pi\rho a^3\int_0^{\chi_n}\sinh^2\chi\,d\chi=4\pi\rho a^3\left(\frac{\sinh 2\chi_n}{4}-\frac{\chi_n}{2}\right)\end{aligned} \tag{12}$$

for the closed and open models. Then use $\chi_n=\eta/2$, the dust-filled solutions, Eq. (8), and Eq. (12) to obtain $M_p=3\tau c^3/4G$ for the open and closed models, as in the flat model.

Also contrary to Schwarzschild space, worldlines of constant $R>R_n$ in the Friedmann dust are spacelike. All particles at $R>R_n$ must be moving towards larger/smaller R in an expanding/collapsing Friedmann universe.

And whereas the event horizons of the Schwarzschild space separate observers in region I/IV from events in region II/I [Fig. 1(a)], the $R=R_n$ sphere (an apparent horizon or Hubble sphere)[12] is not an event horizon. In fact, there are no event horizons in either the flat or open models—all observers will see all events given enough time in these models. The apparent horizon does reside at an event horizon of the closed universe at the point of maximum expansion, but this event horizon (as with the apparent horizon) is dependent on the choice of coordinate origin. All comoving observers must find themselves at the centers of such event and apparent horizons, since space is everywhere isotropic according to the cosmological principle.

But, if we discard the cosmological principle and allow the $R=0$ observer to be at the center of a sphere of Friedmann dust surrounded by Schwarzschild vacuum, the $R_n=2GM/c^2$ analogy is pertinent. In this case, the Schwarzschild mass of the dust sphere is the M found in the Schwarzschild metric. Thus, $2GM/c^2$ is an event horizon of the Schwarzschild space. That is, the Friedmann dust sphere may reside in a black or white hole of the Schwarzschild space. We now show how this adjoining can be accomplished and elaborate on the results.

III. JOINING THE FRIEDMANN AND SCHWARZSCHILD METRICS

Birkoff's theorem[13] tells us the Schwarzschild solution can be adjoined to a dynamical mass distribution, as long as the mass distribution is spherically symmetric. There are two criteria which must be met at the interface (a three-dimensional surface) of the adjoined spacetimes so that the union is a solution of Einstein's equations.[14] First, the metrics must reduce to the same form on the three-dimensional interface. Second, the extrinsic curvature of the surface must be the same when computed with either of the metrics.

To visualize the joining of the spatial portions of the two solutions, remove a sphere (constant radial coordinate r) of vacuum from the Schwarzschild space and replace it with a sphere of dust $\chi=\chi_0$ from the Friedmann space (Fig. 2). As the dust expands (or contracts) into the Schwarzschild space, the value of r at the interface changes, while in comoving $R-W$ coordinates the junction always lies at $\chi=\chi_0$. Since the spherical interface exists in time, the surface is three dimensional in spacetime.

Using the Schwarzschild metric

$$ds^2=-\left(1-\frac{2M}{R}\right)dt^2+\left(1-\frac{2M}{R}\right)^{-1}dr^2+r^2d\Omega^2 \quad (13)$$

and geometrized units ($G=c=1$), the three-dimensional surfaces are given by[15]

$$\frac{t}{2M}=\frac{2}{3}\left(\frac{r}{2M}\right)^{3/2}+2\left(\frac{r}{2M}\right)^{1/2}+\ln\left|\frac{\sqrt{r/2M}-1}{\sqrt{r/2M}+1}\right| + \text{constant}, \quad r=a\chi_0 \quad (14)$$

for the flat model [$a(\tau)$ from flat, dust-filled solution];

$$\frac{t}{2M}=\ln\left|\frac{\beta-\cot\eta/2}{\beta+\cot\eta/2}\right|+\beta[\eta+\alpha(\eta-\sin\eta)] + \text{constant}, \quad r=a\sin\chi_0, \quad (15)$$

where

$$\alpha=\frac{B\sin\chi_0}{4M}, \qquad \beta=\sqrt{2\alpha-1}, \quad (16)$$

for the closed model [$a(\eta)$ from closed, dust-filled solution];

$$\frac{t}{2M}=\ln\left|\frac{\beta-\coth\eta/2}{\beta+\coth\eta/2}\right|+\beta[\eta+\alpha(\sinh\eta-\eta)] + \text{constant}, \quad r=a\sinh\chi_0, \quad (17)$$

where

$$\alpha=\frac{B\sinh\chi_0}{4M}, \qquad \beta=\sqrt{2\alpha+1}, \quad (18)$$

for the open model [$a(\eta)$ from open, dust-filled solution]. Again, M is the Schwarzschild mass generating the Schwarzschild vacuum and $\chi=\chi_0$ describes the interface surface in the Friedmann spacetimes. The angular coordinates of the Schwarzschild spacetime are equal to those of the Friedman spacetimes.

Since the surface is given by $\chi=\chi_0$, the Friedman metric on the surface is simply

$$\begin{aligned} ds^2 &= -a^2\,d\eta^2+a^2\sin^2\chi_0\,d\Omega^2 \\ &= -d\tau^2+a^2\chi_0^2\,d\Omega^2 \\ &= -a^2\,d\eta^2+a^2\sinh^2\chi_0\,d\Omega^2 \end{aligned} \quad (19)$$

for the closed, flat, and open models. In the Schwarzschild spacetime we have from Eq. (14)

$$dt=\sqrt{\frac{r}{2M}}\left(1-\frac{2M}{r}\right)^{-1}dr, \qquad dr=\sqrt{\frac{2M}{r}}\,d\tau, \quad (20)$$

for the flat model; and from Eqs. (15) and (16)

$$dt=\frac{2M\alpha^2(1-\cos\eta)^2\beta\,d\eta}{\alpha(1-\cos\eta)-1}, \qquad dr=\frac{B\sin\chi_0\sin\eta\,d\eta}{2}, \qquad 1-\frac{2M}{r}=\frac{\alpha(1-\cos\eta)-1}{\alpha(1-\cos\eta)}, \quad (21)$$

for the closed model; and from Eqs. (17) and (18)

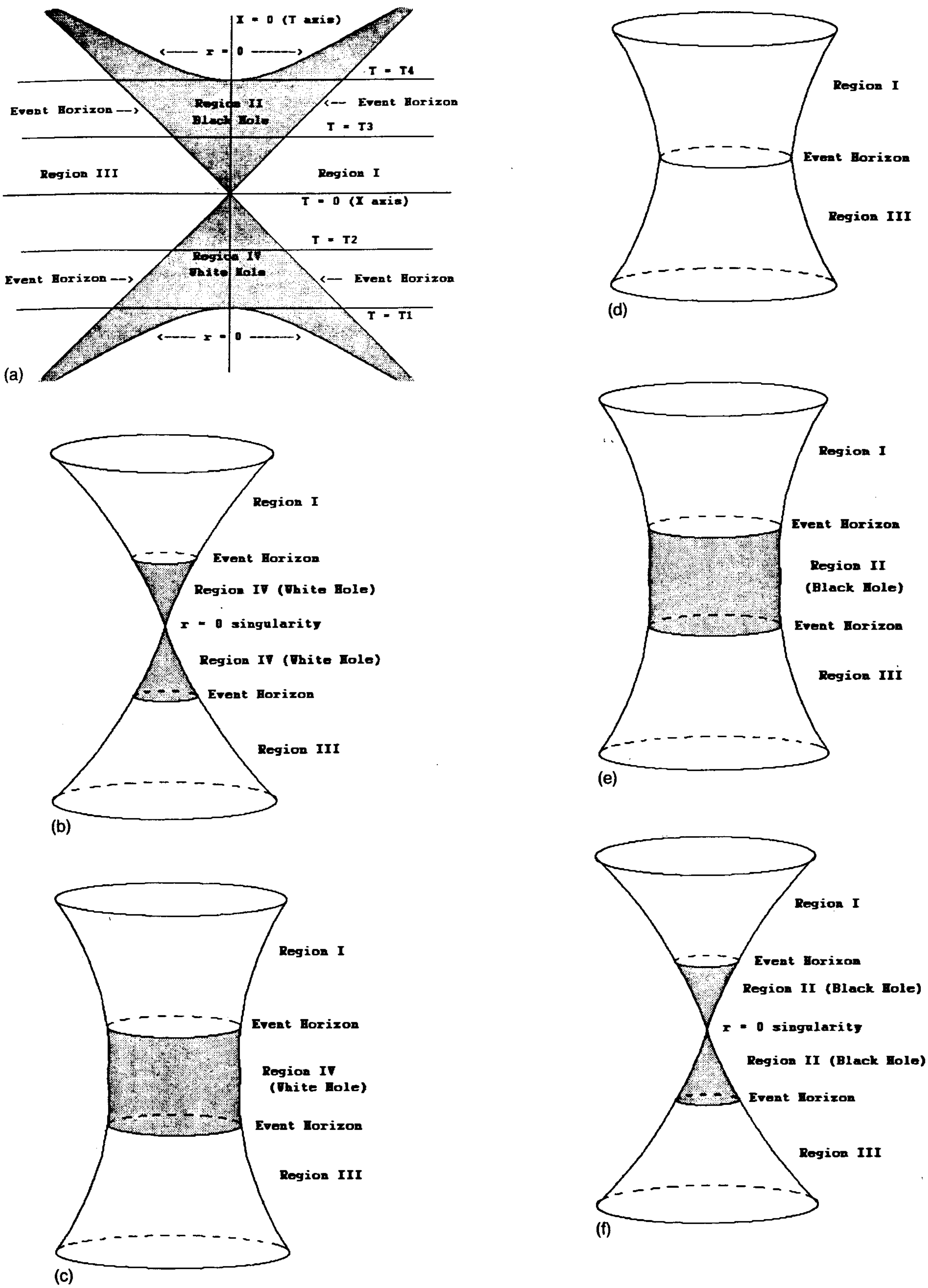

Fig. 1. (a) A sketch of the Kruskal (X,T) plane. (b)–(f) Show a Schwarzschild throat opening and closing between regions I and III and correspond to the various constant T slices shown in this figure. (b) The $T=T1$ slice of the Kruskal (X,T) plane shown in (a). (c) The $T=T2$ slice of the Kruskal (X,T) plane shown in (a). (d) The $T=0$ slice of the Kruskal (X,T) plane shown in (a). (e) The $T=T3$ slice of the Kruskal (X,T) plane shown in (a). (f) The $T=T4$ slice of the Kruskal (X,T) plane shown in (a).

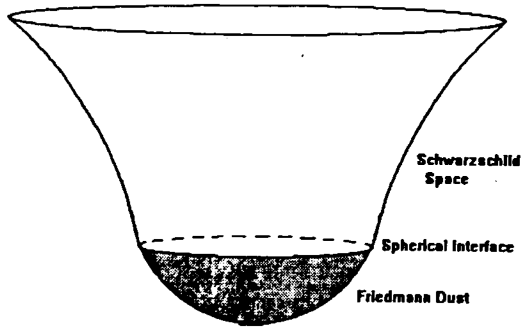

Fig. 2. Friedmann dust (closed model) inside the spherical interface replacing the interior region of the Schwarzschild vacuum.

$$dt=\frac{2M\alpha^2(\cosh\eta-1)^2\beta\, d\eta}{\alpha(\cosh\eta-1)-1},$$

$$dr=\frac{B\sinh\chi_0\sinh\eta\, d\eta}{2}, \tag{22}$$

$$1-\frac{2M}{r}=\frac{\alpha(\cosh\eta-1)-1}{\alpha(\cosh\eta-1)},$$

for the open model. Equations (14) and (20) give

$$-\left(1-\frac{2M}{r}\right)dt^2+\left(1-\frac{2M}{r}\right)^{-1}dr^2+r^2\, d\Omega^2$$
$$=-d\tau^2+a^2\chi_0^2\, d\Omega^2, \tag{23}$$

for the metric of the interface surface in Schwarzschild spacetime (flat model). Equations (15) and (21) give

$$-\left(1-\frac{2M}{r}\right)dt^2+\left(1-\frac{2M}{r}\right)^{-1}dr^2+r^2\, d\Omega^2$$
$$=-a^2\, d\eta^2+a^2\sin^2\chi_0\, d\Omega^2, \tag{24}$$

for the metric of the surface in Schwarzschild spacetime (closed model). Equations (17) and (22) give

$$-\left(1-\frac{2M}{r}\right)dt^2+\left(1-\frac{2M}{r}\right)^{-1}dr^2+r^2\, d\Omega^2$$
$$=-a^2\, d\eta^2+a^2\sinh^2\chi_0\, d\Omega^2, \tag{25}$$

for the metric of the surface in Schwarzschild space-time (open model). Equations (23)–(25) agree with Eq. (19), so we have satisfied the first junction condition for joining the two solutions.

We must now compute the extrinsic curvature of the surface according to each of the solutions. We begin by finding the unit normal vector to the surface according to the Friedman solutions. We have

$$\hat{n}=a^{-1}\frac{\partial}{\partial\chi} \tag{26}$$

since

$$\begin{aligned}
&\hat{n}\cdot\hat{n}=a^{-2}g_{33}=1,\\
&\hat{n}\cdot\frac{\partial}{\partial\theta}=a^{-1}g_{31}=0,\\
&\hat{n}\cdot\frac{\partial}{\partial\phi}=a^{-1}g_{32}=0,\\
&\hat{n}\cdot\frac{\partial}{\partial\eta}=a^{-1}g_{30}=0,\\
&\hat{n}\cdot\frac{\partial}{\partial\tau}=a^{-1}g_{30}=0,
\end{aligned} \tag{27}$$

where we have labeled $x^0=\eta$ (open and closed models), $x^0=\tau$ (flat model), $x^1=\theta$, $x^2=\phi$, $x^3=\chi$. We now compute the extrinsic curvature of the surface according to the Friedman solutions. In all the Friedman models

$$\nabla_0\hat{n}=\frac{\partial(a^{-1})}{\partial x^0}\frac{\partial}{\partial\chi}+a^{-1}\Gamma^\alpha_{03}\mathbf{e}_\alpha. \tag{28}$$

The only nonzero Γ^α_{03} is

$$\Gamma^3_{30}=\frac{1}{a}\frac{\partial a}{\partial x^0} \tag{29}$$

so

$$\nabla_0\hat{n}=0=-K^i_0\mathbf{e}_i,\quad i=0,1,2 \tag{30}$$

or $K^0_0=K^1_0=K^2_0=0$ in all Friedmann models. To find K^i_1 we compute

$$\nabla_1\hat{n}=\frac{\partial a^{-1}}{\partial\theta}\frac{\partial}{\partial\chi}+\frac{1}{a}\Gamma^\alpha_{31}\mathbf{e}_\alpha. \tag{31}$$

The only nonzero Γ^α_{31} is

$$\begin{aligned}
\Gamma^1_{31}&=\frac{\cos\chi}{\sin\chi}\\
&=\frac{1}{\chi}\\
&=\frac{\cosh\chi}{\sinh\chi}
\end{aligned} \tag{32}$$

in the closed, flat, and open models, so

$$\begin{aligned}
K^1_1&=\frac{-\cos\chi_0}{a\sin\chi_0}\\
&=\frac{-1}{a\chi_0}\\
&=\frac{-\cosh\chi_0}{a\sinh\chi_0}
\end{aligned} \tag{33}$$

and $K^0_1=K^2_1=0$ for all models. To find K^i_2 we compute

$$\nabla_2\hat{n}=\frac{\partial a^{-1}}{\partial\phi}\frac{\partial}{\partial\chi}+\frac{1}{a}\Gamma^\alpha_{32}\mathbf{e}_\alpha. \tag{34}$$

The only nonzero Γ^α_{32} is Γ^2_{32} which equals Γ^1_{31}, therefore $K^2_2=K^1_1$ and $K^0_2=K^1_2=0$ in all models.

We next compute the extrinsic curvature according to the Schwarzschild metric. Again we start by finding the unit normal vector to the surface, but this time we need it represented in the Schwarzschild coordinate basis. We know that $\hat{n}\cdot(\partial/\partial\theta)=\hat{n}\cdot(\partial/\partial\phi)=0$ from Eq. (27). Thus, $\hat{n}$ has only r and t components, so

$$\hat{n}=n_t\frac{\partial}{\partial t}+n_r\frac{\partial}{\partial r}. \tag{35}$$

To find n_t and n_r, use

$$\hat{n}\cdot\hat{n}=-\left(1-\frac{2M}{r}\right)n_t^2+\left(1-\frac{2M}{r}\right)^{-1}n_r^2=1 \tag{36}$$

and

$$\hat{n}\cdot\frac{\partial}{\partial x^0}=\frac{\partial r}{\partial x^0}\left(1-\frac{2M}{r}\right)^{-1}n_r-\frac{\partial t}{\partial x^0}\left(1-\frac{2M}{r}\right)n_t=0 \tag{37}$$

to obtain

$$n_t=\sqrt{\frac{2M}{r}}\left(1-\frac{2M}{r}\right)^{-1}\sqrt{\frac{1+\cos\eta}{2}} \tag{38}$$

for all models, and

$$\begin{aligned} n_r&=\cos\chi_0\\ &=1\\ &=\cosh\chi_0. \end{aligned} \tag{39}$$

To find K_0^i, compute

$$\nabla_0\hat{n}=\frac{\partial n_t}{\partial x^0}\frac{\partial}{\partial t}+\frac{\partial n_r}{\partial x^0}\frac{\partial}{\partial r}+n_t\nabla_0\frac{\partial}{\partial t}+n_r\nabla_0\frac{\partial}{\partial r} \tag{40}$$

using

$$\begin{aligned}\nabla_0\frac{\partial}{\partial t}&=\frac{\partial t}{\partial x^0}\nabla_t\frac{\partial}{\partial t}+\frac{\partial r}{\partial x^0}\nabla_r\frac{\partial}{\partial t}\\ &=\frac{\partial t}{\partial x^0}\Gamma^\alpha_{tt}\mathbf{e}_\alpha+\frac{\partial r}{\partial x^0}\Gamma^\alpha_{tr}\mathbf{e}_\alpha\end{aligned} \tag{41}$$

(where r and t are used in subscripts and superscripts to denote r and t components),

$$\begin{aligned}\nabla_0\frac{\partial}{\partial r}&=\frac{\partial t}{\partial x^0}\nabla_t\frac{\partial}{\partial r}+\frac{\partial r}{\partial x^0}\nabla_r\frac{\partial}{\partial r}\\ &=\frac{\partial t}{\partial x^0}\Gamma^\alpha_{tr}\mathbf{e}_\alpha+\frac{\partial r}{\partial x^0}\Gamma^\alpha_{rr}\mathbf{e}_\alpha\end{aligned} \tag{42}$$

and the only nonzero, relevant Christoffel symbols

$$\begin{aligned}\Gamma^t_{rt}&=\frac{M}{r^2}\left(1-\frac{2M}{r}\right)^{-1},\\ \Gamma^r_{rr}&=\frac{-M}{r^2}\left(1-\frac{2M}{r}\right)^{-1},\\ \cancel{\Gamma}&=\frac{M}{r^2}\left(1-\frac{2M}{r}\right)=\Gamma^r_{tt}\end{aligned} \tag{43}$$

to obtain $K_0^i=0$ for all models, in agreement with the Friedmann result. To find K_1^i, compute

$$\nabla_1\hat{n}=\frac{\partial n_t}{\partial\theta}\frac{\partial}{\partial t}+\frac{\partial n_r}{\partial\theta}\frac{\partial}{\partial r}+n_t\nabla_1\frac{\partial}{\partial t}+n_r\nabla_1\frac{\partial}{\partial r} \tag{44}$$

using the only nonzero, relevant Christoffel symbol

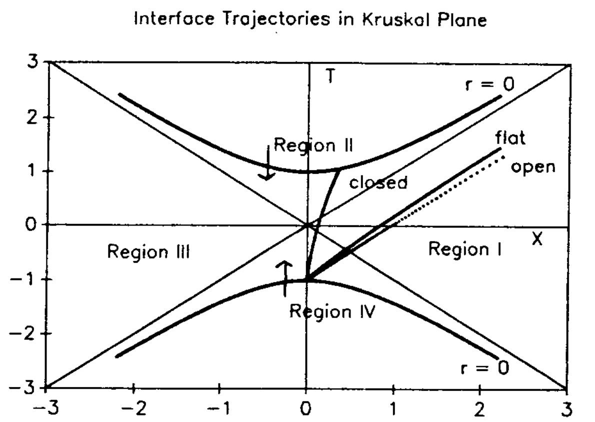

Fig. 3. Example paths taken by the interface surfaces of each of the three dust-filled models in the Kruskal (X,T) plane.

$$\Gamma^1_{r1}=\frac{1}{r} \tag{45}$$

to obtain

$$\begin{aligned}K_1^1&=\frac{-\cos\chi_0}{r}\\ &=\frac{-1}{r}\\ &=\frac{-\cosh\chi_0}{r}\end{aligned} \tag{46}$$

and $K_1^0=K_1^2=0$, in agreement with the Friedmann results of Eq. (33). Finally to find K_2^i, compute

$$\nabla_2\hat{n}=\frac{\partial n_t}{\partial\phi}\frac{\partial}{\partial t}+\frac{\partial n_r}{\partial\phi}\frac{\partial}{\partial r}+n_t\nabla_2\frac{\partial}{\partial t}+n_r\nabla_2\frac{\partial}{\partial r} \tag{47}$$

using $\Gamma^2_{r2}=\Gamma^1_{r1}$ to obtain $K_2^i=K_1^i$, in agreement with the Friedmann results above. Therefore, Eqs. (14)–(18) describe the connection between Friedmann dust and Schwarzschild vacuum.

IV. RESULTS AND CONCLUSION

The interface surfaces (one for each of the Friedmann models) are curves in the Kruskal diagram of Fig. 3. A point on the curve represents a sphere, since the angular coordinates are suppressed. The Friedmann dust may replace the Schwarzschild vacuum on either side of the curve shown in Fig. 3. The side of the curve containing the smaller r values of region I at constant Kruskal time T is referred to as the "interior" Schwarzschild space or region. The other side of the sphere is referred to as the "exterior" Schwarzschild space or region.

We also need to specify what is meant by "inside" and "outside" of the sphere. When we say the Friedmann dust or Schwarzschild vacuum lies inside/outside the sphere, we mean the comoving or Schwarzschild radial coordinate of the dust or vacuum immediately adjoining the interface has a smaller/larger value than the interface. Thus, when dust replaces the exterior/interior vacuum, it lies outside/inside the sphere. We now describe different connections between the

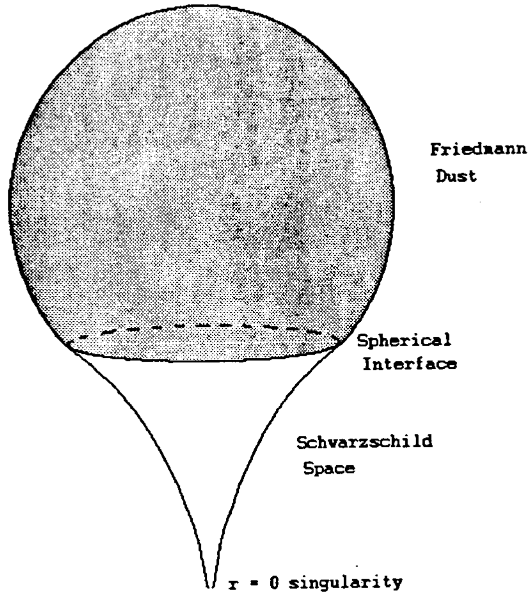

Fig. 4. Friedmann dust (closed model) outside the spherical interface replacing the exterior region of the Schwarzschild vacuum.

various Friedmann dust-filled models and the Schwarzschild vacuum, which lead to an explanation of the black hole universe.

For the closed model, Eq. (39) tells us n_r is negative for $\chi_0>\pi/2$, and positive for $\chi_0<\pi/2$. Since n_r is the r component of $(a^{-1})\partial/\partial\chi$, $\partial/\partial\chi$ points towards smaller r for $\chi_0>\pi/2$ and towards larger r for $\chi_0<\pi/2$. Thus this Friedmann dust replaces the exterior Schwarzschild space if the connection is made at $\chi_0>\pi/2$, and the interior Schwarzschild space if the connection is made at $\chi_0<\pi/2$ (Figs. 2 and 4). There is no such restriction on the flat and open models, so they may replace the interior or exterior region at any χ_0 (Figs. 5 and 6). In all models when the Friedmann dust replaces the interior region, it replaces region III of the Kruskal extension, thus precluding the possibility of a Schwarzschild throat[16] [Figs. 1(b)–1(f)] between regions I and III.

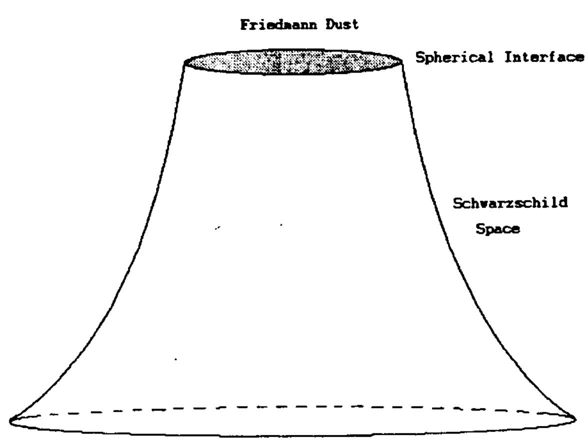

Fig. 5. Friedmann dust (flat model) inside the spherical interface replacing the interior region of the Schwarzschild vacuum.

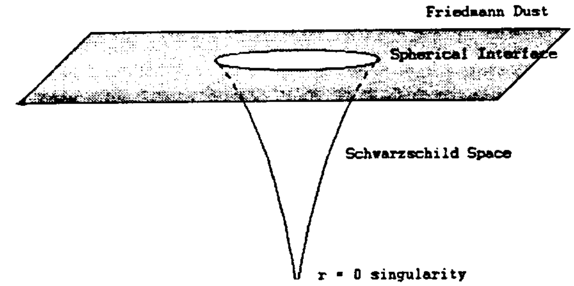

Fig. 6. Friedmann dust (flat model) outside the spherical interface replacing the exterior region of the Schwarzschild vacuum.

When the Friedmann dust replaces the exterior region, the Schwarzschild vacuum must connect it to a mass M, otherwise the interior metric is Minkowskian (Birkoff's theorem). As shown previously, M is equal to the Schwarzshild mass of the Friedmann dust inside $\chi=\chi_0$.

To maintain the possibility of a Schwarzschild throat, the mass M must reside in the singularity at $r=0$. Further, the mass must have resided in the singularity since the Big Bang, or its existence would have precluded the existence of the Kruskal region III. Thus, with the dust outside the sphere, it is as if the Schwarzschild mass at $r=0$ is Friedmann dust not ejected from the Big Bang singularity, and the Schwarzschild vacuum is serving as an "umbilical cord" between the ejected dust and the unejected dust (Figs. 4 and 6).

With this Friedmann–Schwarzschild cosmology model, we see in what sense the Friedmann dust (today in the form of galaxies) may be said to reside in a black or white hole. If the Friedmann dust replaces the interior region (and therefore lies inside the sphere), it can expand out of the initial singularity and out of a white hole of the exterior space (Fig. 3). The closed model will then collapse to its final singularity (Big Crunch) inside the black hole of the exterior space. The open and flat models will forever expand into the exterior space.

If the Friedmann dust replaces the exterior region (and therefore lies outside of the sphere), then the Schwarzschild throat opens between regions I and III. For $T<0$, region III lies through the white hole horizon of region I and vice versa [Fig. 1(c)]. For $T>0$, region III lies through the black hole horizon of region I and vice versa [Fig. 1(e)]. One may say the Friedmann dust lies inside the white or black hole of region III. (The converse statement is equally true, of course.) Thus when the dust replaces the exterior region, it is possible to have expanding Friedmann dust inside a black hole (Fig. 7).

To summarize and conclude, portions of Friedmann dust may be connected to portions of Schwarzschild vacuum. The adjoining may be done with the Friedmann dust inside or outside of the spherical junction, replacing the interior or exterior Schwarzschild space. When the Friedmann dust replaces the interior Schwarzschild space, it may expand out of a white hole and collapse into a black hole of the exterior Schwarzschild space. When the Friedmann dust replaces the exterior Schwarzschild space, it may be connected via a Schwarzschild throat to the Kruskal extension of the Schwarzschild space. Constant T slices of the Kruskal spacetime diagram depict expanding Friedmann dust inside the white and black hole of region III. This diagram (Fig. 3) also

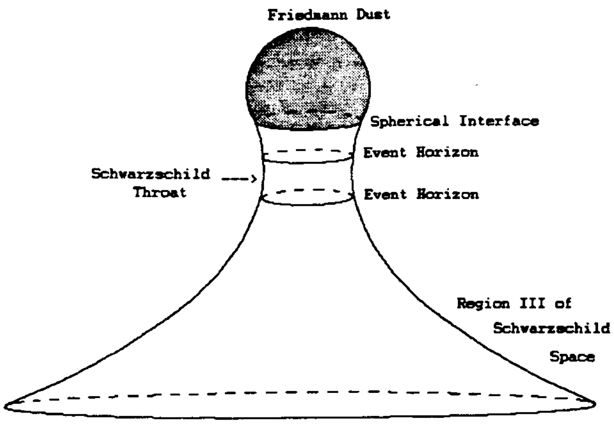

Fig. 7. Friedmann dust (closed model) outside the spherical interface replacing the exterior region of the Schwarzschild vacuum. Here, the Schwarzschild throat has opened to connect regions I and III of the extended Schwarzshild space. The region between the event horizons may be either a black or white hole.

shows the closed model collapsing into the black hole of regions I and III. In this manner, if it is part of a Friedmann dust, the observable universe may be inside a black or white hole.

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UNITS: THE PROUT

A unit of nuclear binding energy equal to one twelfth of the binding energy of the deuteron. Its value is 0·185 MeV or 195×10^{-6} amu. The unit was suggested by Witmer in 1947, because the binding energies of most nuclei are frequently equal to some integral of this value. Heavy nuclei have binding energies of the order of 42 prouts, but the energies of light nuclei can be greater than this. The unit is named after the Scottish physicist William Prout (1786–1850) who put forward a theory that all atoms were composed of hydrogen atoms. Neither the name nor the unit has been widely adopted.

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