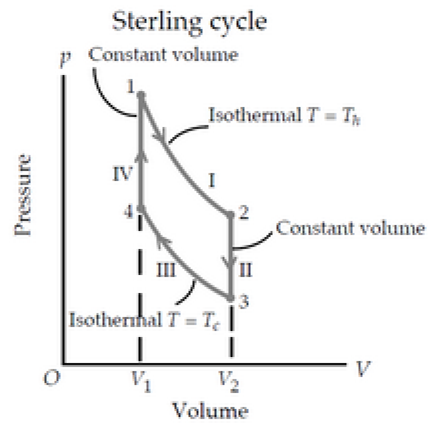
Problem 3.

A P-V Diagram of the Stirling cycle described in problem 3 is shown to the right.



a.) In each leg, I, II, III, and IV, we wish to calculate the heat flows to the cycle during each leg. During the idealized cycle, the working substance is to be in contact with either the hot or the cold reservoir at a given time.

In leg I:

$\Delta U = W + Q$ where U is the energy of the working substance.

Since the temperature is constant during an isothermal process, the internal energy is also held constant $\rightarrow \Delta U = 0 \rightarrow Q = -W$. This is heat being transferred to the working substance from the hot reservoir.

In leg II:

Recall that during an isochoric process: $Q = mC_v\Delta T = m\Delta T(\frac{5R}{2})$. This is heat transferred to the cold reservoir. ΔT here is equal to $T_h - T_c$.

In leg III:

$Q = -W$ by the arguments made in leg I. This is heat transferred to the cold reservoir.

In leg IV:

$Q = m\Delta T(\frac{5R}{2})$ by the arguments in leg II. This is heat from the source being transferred to the working substance. ΔT here is equal to $T_c - T_h$.

b.) What work is done by the cycle during each leg?

In leg I:

$$W_{1\rightarrow 2} = \int_{V_{max}}^{V_{min}} P\,dV = nRT\,ln(\frac{V_{max}}{V_{min}})$$

In leg II:

$W_{2\to3} = 0 \; since \; dV = 0$

In leg III:

This is similar to leg I, but the bounds of our integral are swapped.

$$W_{3\to4} = \int_{V_{min}}^{V_{max}} P\,dV = nRT\,ln(\frac{V_{min}}{V_{max}})$$

In leg IV:

$W_{4\to1} = 0 \; since \; dV = 0.$

If the efficiency of the cycle, $\eta = W/Q_{pos}$, where Q_{pos} is total positive heat flow to the engine, what is the efficiency of the cycle when T_h is 700 K, T_c is 400 K, V_i is 0.5 L, and V_f is 1.5 L. Using the results above, we may calculate the work done during the cycle:

$$W = nR(T_h - T_c)ln(\frac{V_f}{V_i})$$

$$W = (0.1\ mol)(8.314\ \frac{J}{mol * K})(700K - 400\ K)ln(\frac{1.5\ L}{0.5\ L}) = 274.016\ Joules$$

When calculating total positive heat flow to the engine, it is important to note that unlike a Carnot cycle where heat flows discontinuously (flows to system only during isothermal processes), heat flows to the gas of a Stirling engine both during isothermal expansion and isochoric heating.

$$Q_{pos} = C_V(T_h - T_c) + nRT_h ln(\frac{V_f}{V_i})$$

$$Q_{pos} = (5/2 * 8.314\ \frac{J}{mol * K})(700\ K - 400\ K) + (0.1\ mol)(8.314\ \frac{J}{mol * K})(700K)ln(\frac{1.5\ L}{0.5\ L})$$

$$Q_{pos} = 6874.87\ Joules$$

Therefore, by the expression for η, we may find the efficiency of the ideal Stirling cycle.

$$\eta = 274.016\ Joules / 6874.87\ Joules = .039858$$

A carnot cycle operating between the same temperature extrema renders an efficiency through the formula for efficiency for Carnot Cycles:

$$\eta = \frac{W}{Q_H} = 1 - \frac{T_c}{T_h}$$

$$\Rightarrow \eta = 1 - (400\ J / 700\ J) = .4286$$