Since we are far from having the general stationary vacuum solution of Einstein's equation in explicit form,[4] the proof of the uniqueness of the Kerr black holes has proceeded by a relatively long chain of arguments. Logically, the first step—although, historically, one of the last steps—in this chain is the proof by Hawking that the two-dimensional surface formed by the intersection of the horizon of a stationary black hole with a Cauchy surface must have topology S^2. This is established by showing that if it had any other topology, it would be possible to deform it outward into $J^-(\mathscr{I}^+)$ such that the expansion, θ, of the outgoing null geodesics satisfies $\theta \leq 0$ everywhere. This would contradict proposition 12.2.4. Details of the proof can be found in Hawking and Ellis (1973). (See also Gannon 1976 for some results in the nonstationary case.)

The next step in the uniqueness proof, also due to Hawking, is the demonstration that a stationary vacuum black hole must be static or axisymmetric. First, we note that in a stationary spacetime containing a black hole, the time translation isometry must leave the horizon invariant. Hence, the Killing field ξ^a must lie tangent to the horizon and, hence, always must be spacelike or null on the horizon. Now, one of the following three possibilities must hold: (i) No ergosphere is present in the spacetime, i.e., the stationary Killing field ξ^a is everywhere timelike or null outside the black hole. In this case, ξ^a must be null on the horizon. (ii) An ergosphere is present but is disjoint from the horizon, and ξ^a is null on the horizon. (iii) An ergosphere is present and intersects the horizon, as happens for a Kerr black hole. In this case, ξ^a is spacelike on (a portion of) the horizon. In case (i), a generalization of a theorem of Lichnerowicz (1955) establishes that the spacetime must be static (Hawking and Ellis 1973). Under some additional assumptions, results of Hajicek (1973) show that the outer boundary of the ergosphere in a stationary vacuum spacetime always must intersect the horizon. Thus, it appears that case (ii) cannot occur. Plausibility arguments against case (ii) also are given in Hawking and Ellis (1973). Finally, in case (iii), using the properties of the horizon in a stationary spacetime and using the analyticity of stationary vacuum solutions (Müller zum Hagen 1970), Hawking proved existence of a one-parameter group of isometries which commute with the stationary isometries and whose orbits on the horizon coincide with the null geodesic generators of the horizon. Thus, one obtains a Killing field χ^a distinct from ξ^a, and by taking a linear combination of χ^a and ξ^a, one obtains a Killing field ψ^a whose orbits are closed, i.e., an axial Killing field.[5] Again, details of this proof are given in Hawking and Ellis (1973).

4. Indeed, until the early 1970s the Kerr solutions were virtually the only known stationary, nonstatic, asymptotically flat vacuum solutions. As discussed in sections 7.1 and 7.4, great progress has been made in obtaining the general stationary, axisymmetric vacuum solution, but even so we are far from having the solutions in sufficiently explicit form to determine if they represent black holes.

5. The proof that a stationary, nonstatic black hole must be axisymmetric continues to hold in the case where a distribution of matter is placed outside a rotating black hole. This leads to an apparent paradox since one would expect it to be possible to "hold in place" a nonaxisymmetric distribution of matter far from the black hole, thereby producing a stationary nonaxisymmetric spacetime. The resolution of this paradox is that such a matter distribution will produce an effective "tidal friction" causing the black hole to "spin down" and thus be nonstationary until it reaches a final static state. A discussion of this process is given by Hawking and Hartle (1972).