ECCENTRIC LOADS ON WELD GROUPS
TABLES XIX–XXVI

ULTIMATE STRENGTH METHOD*

When weld groups are loaded in shear by an external load that does not act through the center of gravity of the group, the load is eccentric and will tend to cause a relative rotation and translation between the parts connected by the weld. The point about which rotation tends to take a place is called the *instantaneous center of rotation*. Its location is dependent upon the eccentricity, geometry of the weld group, and deformation of the weld at different angles of the resultant elemental force relative to the weld axis.

The individual resistance force of each unit weld element can then be assumed to act on a line perpendicular to a ray passing through the instantaneous center and that element's location (see Fig. 1).

The ultimate shear strength of weld groups can be obtained from the load deformation relationship of a single unit weld element which is expressed as:

$$R = R_{ult}(1 - e^{-\mu\Delta})^{\lambda}$$

where

R = Shear force in a single element at any given deformation
R_{ult} = Ultimate shear load of a single element
μ, λ = Regression coefficients
Δ = Deformation of a weld element
e = Base of natural logarithm $\approx$ 2.718

Unlike the load-deformation relationship for bolts, strength and deformation performance in welds are dependent on the angle Θ that the resultant elemental force makes with the axis of the weld element (see Fig. 1).

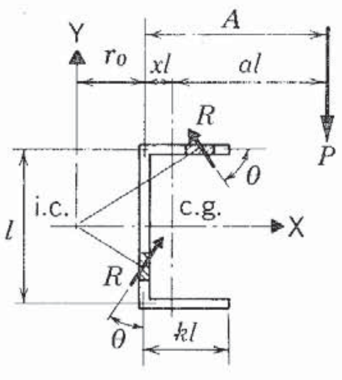


Fig.1

The critical weld element is usually (but not always) the weld element furthest from the instantaneous center. The critical deformation can be calculated as:

$$\Delta_{max} = 0.225\ (\Theta + 5)^{-0.47}, \text{ where } \Theta \text{ is expressed in degrees}$$

* *Butler, Pal and Kulak* Eccentrically Loaded Weld Connections. *ASCE Journal of the Structural Division, Vol. 98, No. ST5, May 1972 (pp. 989-1005).*

The deformation of other weld elements can then be calculated as:

$$\Delta = \frac{r}{r_{max}} \Delta_{max}$$

The values of R_{ult}, μ and Δ depend on the value of the angle Θ and can be obtained from the following relations:

$$R_{ult} = \frac{10 + \Theta}{0.92 + 0.0603\Theta}$$

$$\mu = 75\, e^{0.0114\,\Theta}$$

$$\lambda = 0.4\, e^{0.0146\,\Theta}$$

The total resistance of all the weld elements combine to resist the eccentric ultimate load, and if the correct location of the instantaneous center has been selected, the three equations of statics will be satisfied. General performance curves for values of $\Theta = 0°$, $10°$, $30°$ and $90°$ are shown in Fig. 2.

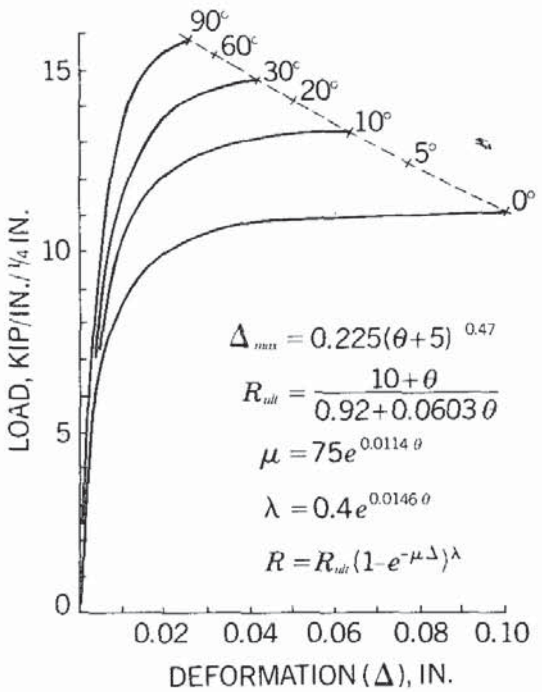


Fig. 2

TABLES XIX-XXVI

To obtain reliable coefficients based on ultimate strength analysis that would replace the traditional elastic C-value in eccentric load design tables, several intermediate steps are required. These include proper correlation factors applied to research data, the application of an acceptable factor of safety, and the use of upper bound limits at points of critical stress in the group to prevent overstress in the weld metal.

Tests were performed on eccentrically loaded ¼-in. fillet weld groups made with E60XX electrodes. To obtain C-tables for E70XX electrode series and a base weld size of 1⁄16 in., the ultimate capacities were adjusted by the factor of ¼ × 70/60. The resulting value was then converted to an allowable stress by multiplying it by 0.30.

Additionally, this value was reduced by a factor that would prevent the stress in any element of weld to exceed the allowable stress for fillet weld metal as required in AISC ASD Specification Sect. J2.4. Tests have demonstrated that the fusion face of weld metal and base material is not critical in determining weld strength. The Manual tables are, therefore, valid for weld metal with a strength level that matches the base material.

To obtain the capacity of a weld group carrying an eccentric load:

$$P = CC_1Dl$$

where

P = Allowable load, kips
C = Tabular value
C_1 = Coefficient for electrode used (see table below)
= 1.0 for E70XX electrodes
l = Length of vertical weld, in.
D = Number of sixteenths of an inch, weld size

Electrode	E60	E70	E80	E90	E100	E110
F_v (ksi)	18.0	21.0	24.0	27.0	30.0	33.0
C_1	0.857	1.0	1.14	1.29	1.43	1.57

Tables XIX through XXVI are based on welds made with E70XX electrodes and matching base metal (AWS Table 4.1.1). They also recognize that for equal leg fillet welds the area of the fusion surface is always larger than the leg dimension times the weld length; therefore, the values are based upon the strength through the throat of the weld ($0.3 \times F_u \times 0.707 \times 1/16$). When electrodes other than E70XX are used with matching or stronger base metals, multiply by C_1 values tabulated in the table above.

These Manual tables may be easily extended to inclined eccentric loads through Alternate Method 2 described later.

ALTERNATE METHOD 1—ELASTIC

In addition to the ultimate strength method previously described, the elastic method may be used to design/analyze eccentrically loaded weld groups not conforming to the AISC Manual tables. By assuming each weld element as a line coincident with the edge of a fillet weld, each unit element is assumed to support:

1. An equal share of the vertical component of the load.
2. An equal share of the horizonal component of the load.
3. A proportional share (dependent on the element's distance from the centroid of the group) of the eccentric moment portion of the load.

The maximum load is determined from the vectorial resolution of these stresses at the element most remote from the group's centroid. This elastic method, although providing a simplified and conservative approach, does not render a consistent factor of safety and in some cases results in excessively conservative designs of connections.

ALTERNATE METHOD 2

As discussed in the eccentrically loaded bolt section, this new method permits extension of the published Manual weld tables to eccentric loads that are inclined at an angle Θ from the vertical. It is based on arithmetic, rather than vectorial, addition of connector strength as an exaggerated load effect or, equivalently, on a linear interaction between eccentric and direct shear.*

First, define C_{max} as the maximum concentric weld coefficient (e.g. 0.928 (1 + 2k) for C-welds).

Next, let

$$A = \frac{C_{max}}{C_o} \geq 1.0$$

where C_o is the AISC Manual tabulated C for a given vertical load case. For a particular connector pattern and load eccentricity distance, A is a constant relative to the load angle Θ; it serves as the single characteristic input property of the connector geometry.

The approximate eccentricity coefficient C_a for the inclined load is

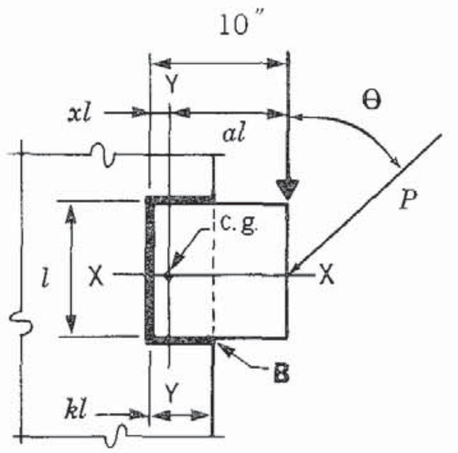
$$\frac{C_a}{C_o} = \frac{A}{(\sin\Theta + A\cos\Theta)} \geq 1.0$$

Only this one equation (the same as for bolts) is necessary to represent capacity as a function of load angle. The allowable load P is determined as previously described except that the computed C_a replaces the tabulated C value. For more guidance and a graphical design aid on application of this approximate method, see bolt discussion on Alternate Method 2.

EXAMPLE 26

Given:

C-Weld group as shown with $l = 10$ in., $kl = 5$ in. and $xl + al = 10$ in. Find the maximum allowable load P ($\Theta = 0$) for a ⅜-in. weld using E70XX electrodes by using Table XXIII.



* *Iwankiw, Nestor R.* Design for Eccentric and Inclined Loads on Bolt and Weld Groups *AISC Engineering Journal, 4th Qtr., 1987, Chicago, Ill.*

** *Iwankiw, Nestor R.* Addendum/Closure on Design for Eccentric and Inclined Loads on Bolt and Weld Groups *AISC Engineering Journal, 3rd Qtr., 1988, Chicago, Ill.*

Solution:

$$k = \frac{kl}{l} = \frac{5}{10} = 0.5;$$

Enter Table XXIII: For $k = 0.5$, $x = 0.125$

$xl = 0.125 \times 10 = 1.25$ in.
$al = 10 - xl = 8.75$ in.; $a = 0.875$

Interpolating between $a = 0.8$ and $a = 0.9$ for $k = 0.5$:

$C = 0.704$
$D = 6$ (⅜-in. weld)
$C_1 = 1.0$ for E70XX electrodes (see table above)
$P = C_1CDl = 1.0 \times 0.704 \times 6 \times 10 = 42.2$ kips

EXAMPLE 27

Given:

C-Weld as shown in Ex. 26, with $l = 10$ in., $kl = 5$ in., and $al = 0.875$, except eccentric service load $P = 90$ kips at 75.° Determine minimum required weld size.

Use Alternate Method 2:

Solution:

From Table XXIII (as in Ex. 26)

$$C_o = C = 0.704$$

$$C_{max} = 0.928\,(1 + 2(.5)) = 1.856$$

$$A = \frac{1.856}{0.704} = 2.64$$

$$\frac{C_a}{C_o} = \frac{2.64}{[.966 + 2.64\,(.259)]} = 1.6 \geq 1.0, \quad \textbf{o.k.}$$

$$C_a = 1.6\,(0.704) = 1.13$$

$$D = \frac{90}{(1.13)\,10} = 7.96, \text{ say 8, use ½-in weld}$$

Using Alternate Method 1 (elastic), $C_e = 1.09 < 1.13$

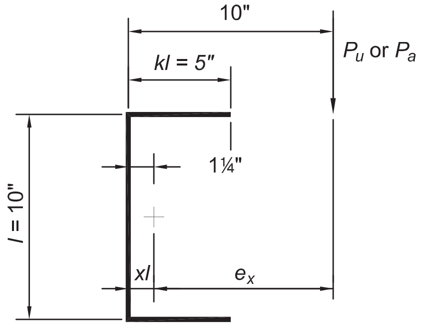
EXAMPLE II.A-26 ECCENTRICALLY LOADED WELD GROUP (IC METHOD)

Given:

Determine the largest eccentric force, acting vertically and at a 75° angle, that can be supported by the available shear strength of the weld group, using the instantaneous center of rotation method. Use a ⅜-in. fillet weld and 70-ksi electrodes. Use AISC *Manual* Table 8-8.

Solution A (θ = 0°):

Assume that the load is vertical (θ = 0°) as shown:



l = 10.0 in.

kl = 5.00 in.

$$k = \frac{kl}{l}$$
$$= \frac{5.00 \text{ in.}}{10.0 \text{ in.}}$$
$$= 0.500$$

$$xl = \frac{(kl)^2}{2(kl)+l}$$
$$= \frac{(5.00 \text{ in.})^2}{2(5.00 \text{ in.})+10.0 \text{ in.}}$$
$$= 1.25 \text{ in.}$$

$$xl + al = 10.0 \text{ in.}$$
$$1.25 \text{ in.} + a(10.0 \text{ in.}) = 10.0 \text{ in.}$$
$$a = 0.875$$

By interpolating AISC *Manual* Table 8-8, with θ = 0°, a = 0.875 and k = 0.500:

$$C = 1.88$$

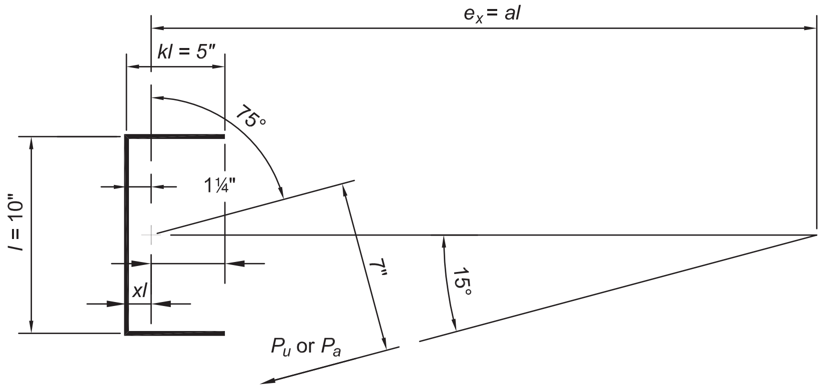
From AISC *Manual* Equation 8-13:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi C C_1 D l$ $= 0.75(1.88)(1.0)(6\text{ sixteenths})(10.0\text{ in.})$ $= 84.6\text{ kips}$ Thus, P_u must be less than or equal to 84.6 kips.	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{C C_1 D l}{\Omega}$ $= \dfrac{1.88(1.0)(6\text{ sixteenths})(10.0\text{ in.})}{2.00}$ $= 56.4\text{ kips}$ Thus, P_a must be less than or equal to 56.4 kips.

Note: The eccentricity of the load significantly reduces the shear strength of this weld group as compared to the concentrically loaded case.

Solution B (θ = 75°):

Assume that the load acts at an angle of 75° with respect to vertical (θ = 75°) as shown:



As determined in Solution A,

$k = 0.500$ and $xl = 1.25$ in.

$$e_x = al$$
$$= \frac{7.00\text{ in.}}{\sin 15°}$$
$$= 27.0\text{ in.}$$

$$a = \frac{e_x}{l}$$
$$= \frac{27.0\text{ in.}}{10.0\text{ in.}}$$
$$= 2.70$$

By interpolating AISC *Manual* Table 8-8, with θ = 75°, a = 2.70 and k = 0.500:

$$C = 1.99$$

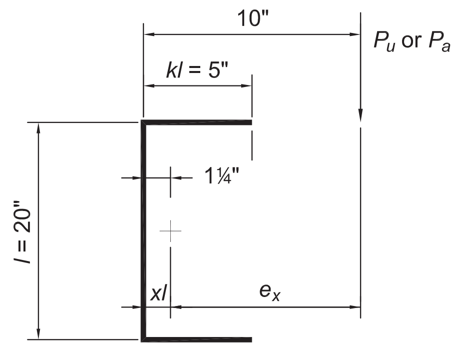
From AISC *Manual* Equation 8-13:

LRFD	ASD
$\phi R_n = \phi C C_1 D l$ $= 0.75(1.99)(1.0)(6 \text{ sixteenths})(10.0 \text{ in.})$ $= 89.6 \text{ kips}$ Thus, P_u must be less than or equal to 89.6 kips.	$\dfrac{R_n}{\Omega} = \dfrac{C C_1 D l}{\Omega}$ $= \dfrac{1.99(1.0)(6 \text{ sixteenths})(10.0 \text{ in.})}{2.00}$ $= 59.7 \text{ kips}$ Thus, P_a must be less than or equal to 59.7 kips.

EXAMPLE II.A-27 ECCENTRICALLY LOADED WELD GROUP (ELASTIC METHOD)

Given:

Determine the largest eccentric force that can be supported by the available shear strength of the welds in the connection, using the elastic method. Compare the result with that of the previous example. Use ⅜-in. fillet welds and 70-ksi electrodes.



Solution:

Direct Shear Force per Inch of Weld

LRFD	ASD
$r_{pux} = 0$ $r_{puy} = \dfrac{P_u}{l}$ (*Manual* Eq. 8-5a) $= \dfrac{P_u}{20.0\text{ in.}}$ $= 0.0500\dfrac{P_u}{\text{in.}}$	$r_{pax} = 0$ $r_{pay} = \dfrac{P_a}{l}$ (*Manual* Eq. 8-5b) $= \dfrac{P_a}{20.0\text{ in.}}$ $= 0.0500\dfrac{P_a}{\text{in.}}$

Additional Shear Force due to Eccentricity

Determine the polar moment of inertia referring to the AISC *Manual* Figure 8-6:

$$
\begin{aligned}
I_x &= \frac{l^3}{12} + 2(kl)(y^2) \\
&= \frac{(10.0\text{ in.})^3}{12} + 2(5.00\text{ in.})(5.00\text{ in.})^2 \\
&= 333\text{ in.}^4/\text{in.}
\end{aligned}
$$

$$
\begin{aligned}
I_y &= \frac{2(kl)^3}{12} + 2(kl)\left(\frac{kl}{2} - xl\right)^2 + l(xl)^2 \\
&= \frac{2(5.00\text{ in.})^3}{12} + 2(5.00\text{ in.})(2.50\text{ in.} - 1\tfrac{1}{4}\text{ in.})^2 + (10.0\text{ in.})(1\tfrac{1}{4}\text{ in.})^2 \\
&= 52.1\text{ in.}^4/\text{in.}
\end{aligned}
$$

$$I_p = I_x + I_y$$
$$= 333 \text{ in.}^4\text{/in.} + 52.1 \text{ in.}^4\text{/in.}$$
$$= 385 \text{ in.}^4\text{/in.}$$

LRFD	ASD
$r_{mux} = \dfrac{P_u e c_y}{I_p}$ (*Manual* Eq. 8-9a) $= \dfrac{P_u(8.75 \text{ in.})(5.00 \text{ in.})}{385 \text{ in.}^4/\text{in.}}$ $= \dfrac{0.114P_u}{\text{in.}}$	$r_{max} = \dfrac{P_a e c_y}{I_p}$ (*Manual* Eq. 8-9b) $= \dfrac{P_a(8.75 \text{ in.})(5.00 \text{ in.})}{385 \text{ in.}^4/\text{in.}}$ $= \dfrac{0.114P_a}{\text{in.}}$
$r_{muy} = \dfrac{P_u e c_x}{I_p}$ (*Manual* Eq. 8-10a) $= \dfrac{P_u(8.75 \text{ in.})(3.75 \text{ in.})}{385 \text{ in.}^4/\text{in.}}$ $= \dfrac{0.0852P_u}{\text{in.}}$	$r_{may} = \dfrac{P_a e c_x}{I_p}$ (*Manual* Eq. 8-10b) $= \dfrac{P_a(8.75 \text{ in.})(3.75 \text{ in.})}{385 \text{ in.}^4/\text{in.}}$ $= \dfrac{0.0852P_a}{\text{in.}}$
Resultant shear force:	Resultant shear force:
$r_u = \sqrt{(r_{pux} + r_{mux})^2 + (r_{puy} + r_{muy})^2}$ (*Manual* Eq. 8-11a) $= \sqrt{\left(0 + \dfrac{0.114P_u}{\text{in.}}\right)^2 + \left(\dfrac{0.0500P_u}{\text{in.}} + \dfrac{0.0852P_u}{\text{in.}}\right)^2}$ $= \dfrac{0.177P_u}{\text{in.}}$	$r_a = \sqrt{(r_{pax} + r_{max})^2 + (r_{pay} + r_{may})^2}$ (*Manual* Eq. 8-11b) $= \sqrt{\left(0 + \dfrac{0.114P_a}{\text{in.}}\right)^2 + \left(\dfrac{0.0500P_a}{\text{in.}} + \dfrac{0.0852P_a}{\text{in.}}\right)^2}$ $= \dfrac{0.177P_a}{\text{in.}}$
Since r_u must be less than or equal to the available strength, from AISC *Manual* Equation 8-2a,	Since r_a must be less than or equal to the available strength, from AISC *Manual* Equation 8-2b,
$r_u = 0.177P_u \le \phi r_n$ $P_u \le \dfrac{\phi r_n}{0.177}$ $\le \dfrac{1.392 \text{ kips/in.}}{\text{sixteenth}}(6 \text{ sixteenths})\left(\dfrac{\text{in.}}{0.177}\right)$ ≤ 47.2 kips	$r_a = 0.177P_a \le r_n/\Omega$ $P_a \le \dfrac{r_n/\Omega}{0.177}$ $\le \dfrac{0.928 \text{ kip/in.}}{\text{sixteeth}}(6 \text{ sixteenths})\left(\dfrac{\text{in.}}{0.177}\right)$ ≤ 31.5 kips

Note: The strength of the weld group predicted by the elastic method, as shown here, is significantly less than that predicted by the instantaneous center of rotation method in Example II.A-26.