What's wrong with this local realistic counter-example to Bell's theorem? It delivers the correct quantum-mechanical statistics for any EPR-Bohm experiment.

JenniT[1]

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Abstract: Combining frames of reference with high-school maths and logic, and allowing that a measurement perturbs the measured system, we offer a local realistic counter-example (*model*, for short) to Bell's theorem. Designated L*R, to distinguish it from naive local realism (NLR), the model yields the correct outcome statistics for EPRB-Bohm experiments, in full accord with quantum mechanics. In further agreement with quantum mechanics, L*R repudiates Bell's theorem by showing that there is no rational basis here for constructing a Bell-inequality. It follows that Bell's theorem is not a constraint on the local realism developed here. Further, built from elements of physical reality, with no abstract entities, L*R minimises the need for semantics. It thus provides a basis for understanding, and meeting (*perhaps*), the popular requirement that local realists should be capable of modelling EPR-Bohm experiments on two classical computers. [That prudent "*perhaps*" reflects this model-maker's caution when attempting to meet any challenge ... especially those that go beyond QM ... before fully understanding it.]

Introduction:

"*Somewhere in our doctrine is a hidden concept, unjustified by experience, which we must eliminate to open up the road*," Born[2]. Wheeler[2] asked us to make all our mistakes as quickly as possible. Feynman[2] looked at negative probabilities. Taking it as a given, that the reader is familiar with the Bell literature, I use *frames of reference* to take local realism to its limit. That limit is identified as L*R to distinguish it from naive local realism (NLR). As in special relativity, frames of reference provide different accounts of the same phenomena (Bell[2], ropes and rockets).

Devoid of abstract concepts, being solely based on accepted elements of physical reality, L*R allows that a measurement perturbs the measured system. L*R is also *realistic* in the sense that it provides values for experiments that cannot be performed, with all such theoretical outcome distributions normalized (summing to unity), and all practical *testable* outcomes normalized and in agreement with QM.

Technically: The source emits an ensemble of pairs, one pair at a time, irrespective of, and unaffected by any test setting chosen by Alice and Bob. Particle responses are given for any orientation. Particle correlations are probed over the ensemble, across the differential settings of the two detectors. Here, though not discussed, each hidden variable is the orientation of total spin (total angular momentum) in 3-space, for each particle. In QM terms, such spins are pair-wise correlated in the spherically symmetric singlet state, with no two pairs the same.

Since these results derive from local realism, I conclude: Bell's Theorem is not an impediment to L*R and its local-realistic view of the world. Further: These results, evidently extending beyond QM in analytic detail (and being wholly based on local realistic considerations) are the reason that I am sometimes reluctant to consider specific examples – whereby we lose some of the beauty, and the important check on each result, that the generality here provides. Also, in my view, too many words sometimes accompany specific examples. So, preferring examples formulated in general terms, that is how I prefer to reply. Nevertheless, and of course: specific numerical examples can be given and will be discussed for any experimental arrangement. I am committed to answer any question re L*R.

A preliminary introduction may be found in the on-line PF thread, commencing with http://www.physicsforums.com/showthread.php?t=475076 The following guidance is offered: Any derivation from L*R, yielding a result contrary to QM, is in error. L*R would not have been offered for consideration had it failed to meet this boundary condition: and to confirm this advice, all such results are fully derived in Appendix A. Results contrary to Bell's theorem are, of course, to be expected; again, in full accord with QM. To put it another way, and be clear: L*R, reflecting all the theoretical and practical boundary conditions required by any local realistic physics, allows Alice and Bob to test all EPR-Bohm quantum-entangled possibilities in theory, delivering all the results that they will find in practice.

Terms and notation:

<u>**a, b, c**</u> = reference-orientations (ROs); rays or half-lines in any 2-space orthogonal to a particle's line-of-flight, they correspond to the detector settings typically associated with Alice and Bob. See also: shadow-orientations (SOs) **a**', **b**', **c**' and Fig. 1.

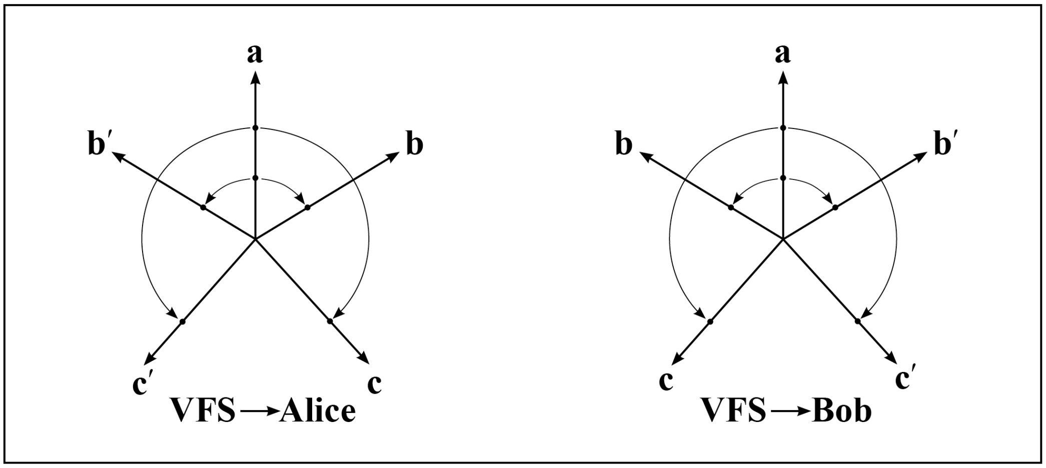

Figure 1: Views from the particle source (VFS), showing detector settings used after orientation **a** is freely chosen; etc.

<u>ab</u> = angle between orientations **a** and **b**; etc.

<u>bi-angle</u> = term used to reflect the varied geometry that L*R must account for. Alice and Bob (savvy Bell-supporters with free will) are allowed to test L*R to its limits; i.e., they conduct tests beyond those commonly described in the Bell literature. Of course, as usual, they each measure outcomes on the customary orientations (**a**, **b**, **c**), one at a time, yielding correlations across one angle (aa, bb, cc, ab, bc, ca) at a time; but we here allow them to test beyond that range, expecting no objections. These additional tests, which L*R must satisfy, are currently seen to be the best way of demonstrating and explaining L*R's power: L*R itself delivering any testable outcome, independent of other tests (as it must).

With these preliminaries, we proceed to understand bi-angles as follows: Let Alice and Bob, first and freely, agree on orientation **a**; then **b**; let them then agree to measure all the ab correlations. Let them then agree to measure (see Fig.1), all the ab' correlations to ensure that P(ab··|ab) = P(ab'··|ab'), where ·· = + +, + –, – + or – –. Next, ac or bc may be chosen from an infinite set of angles. Let them choose ac, and let them proceed to measure all the ac correlations; then all the ac' correlations to ensure that P(ac··|ac) = P(ac'··|ac'). We then have this important fact: Out of that infinity of ab and ac choices, Bob and Alice have also been testing over bc = ac – ab, b'c = ac + ab', bc' = ac' – ab, b'c' = ac' – ab'. That is, Alice and Bob have been testing over bc-designated angles (ignoring the primes) that can take on *two values*, bc = ac ± ab. So <u>bc is then termed a *bi-angle*</u>; etc. Allowing for, and accounting for such stringent testing *across variable geometries*, L*R delivers the average correlation over the bi-angle. Subsequently, whichever specific bi-angle value Bob and Alice select for testing, L*R delivers the correlation for that value. In every case, L*R delivers the correct (QM) outcomes and distributions for any test on any orientation or across any angle: independent of the test choices and any order of testing that Alice and Bob might make.

Table 0 shows some bi-angles, beyond the nominal three angles given in typical EPR-Bohm-Bell scenarios.

Table 0: How L*R accommodates the geometry associated with bi-angles.
In any column, single angles (e.g., **10**, in degrees) show the first two test angles selected by Alice & Bob. Bi-angles are shown thus: 20/40. The last angle selected for testing, shown bold (**20**/40) is the selection that defines the geometry attaching to that set of tests. L*R delivers the correct (QM) outcomes and normalized distributions for any test.

Column #	1	2	3	4	5	6	7	8
ab	**10**	**10**	**10**	**10**	10/**50**	**10**/50	**50**	**50**
ac	**30**	**30**	**10**/30	10/**30**	**30**	**30**	**30**	**30**
bc	**20**/40	20/**40**	**20**	**20**	**20**	**20**	20/**80**	**20**/80

The reader should sketch and compare the geometry of columns 1 & 2. Accommodating that geometry, L*R delivers the *same* correct Pab and Pac correlations; and the differing but correct Pbc results for each. We conclude: Single-angle or two-angle probing over three arbitrary orientations in 2-space yields two geometries: reducing to one geometry when the third angle is defined. The model-maker allows that Alice and Bob are canny enough to test L*R over ab/ab' and ac/ac' under the two geometries defined in columns 1-2; etc. (They would know that the result must be exactly as L*R provides.) Similarly, for the 3-4, 5-6, 7-8 comparisons.

In its tri-partite mode (Table 1 below, which is beyond direct testing), L*R yields the correct results: for all testable orientations; for all testable angles; and, *also beyond testing,* for any two angles, with the average result for the concomitant bi-angles: then, whichever value of the bi-angle is subsequently tested, L*R yields the correct (QM) result for that angle.

<u>Cab</u> = $\cos^2 s\text{ab}$; etc.

<u>HVC</u> = Hidden variable class; short for HV *equivalence class*, defined by the outcomes (+/–) that would be delivered after measurement interactions at specific orientations or combinations thereof.

<u>L*R</u> = symbol denoting the version of local realism that is used here, in conjunction with frames of reference (ROs) and perturbative measurement interactions.

<u>P(a+|**a**)</u> = 1/2 (by summation over relevant classes) = probability for outcome a+ given RO **a**. Likewise P(a–|**a**) = 1/2; etc. See (A4).

<u>P(ab++|ab)</u> = Probability for outcome ab++ across **a** and **b**; i.e., probability of the combined outcome **a** = +, **b = +**, across the clearly-defined (unambiguous) angle ab; etc. See (A1a').

<u>P(ab++|abc)</u> = Probability for outcome ab++ across **a**, **b**, **c** IF they could be physically tested; i.e., probability of the combined outcome **a** = +, **b** = +, across the three orientations when L*R is mathematically tested (beyond experiment); etc. See (A1a).

<u>RO</u> = reference orientation; any allowable setting of a detector. Orientations are here limited to the 2-space orthogonal to a particle's line-of-flight. In Fig. 1, the RO is **a**.

<u>*s*</u> = intrinsic particle spin.

<u>Sab</u> = $\sin^2 s\text{ab}$; etc.

SO = shadow orientation (SOs); rays or half-lines in any 2-space orthogonal to a particle's line-of-flight, they correspond to additional detector settings beyond the **a**, **b**, **c** typically associated with Alice and Bob. See Fig. 1.

VFS = view from source (Fig. 1); to be clear about the relations between Alice's and Bob's settings.

Analysis:

Table 1 is testable, part-by-part, by real tests; and wholly by mathematical tests. derived as the average over Tables A1-A3, Table 1 yields the correct (QM) result for any test; *that is its purpose*. Table 2 (fully testable and directly yielding correct QM results) derives from calculations detailed in Appendix A. Tables and equations beginning with 'A' are to be found in Appendix A.

Table 1: The L*R Model.

Hidden Variable Class (HVC) from +/– outcomes on **a**, **b**, **c**.			Normalized HVC Distribution (sum = 1) over three Reference Orientations (**a**, **b**, **c**).
HVC ID#	HV→Alice **a b c**	HV→Bob **a b c**	NB: The probabilities below, simultaneously valid, are not simultaneously measureable.
01	+ + +	– – –	P1: P(01|abc) = [Cab.Cac + Cab.Cbc + Cac.Cbc]/6.
02	+ + –	– – +	P2: P(02|abc) = [Cab.Sac + Cab.Sbc + Sac.Sbc]/6.
03	+ – +	– + –	P3: P(03|abc) = [Sab.Cac + Sab.Sbc + Cac.Sbc]/6.
04	+ – –	– + +	P4: P(04|abc) = [Sab.Sac + Sab.Cbc + Sac.Cbc]/6.
05	– + +	+ – –	P5: P(05|abc) = [Sab.Sac + Sab.Cbc + Sac.Cbc]/6.
06	– + –	+ – +	P6: P(06|abc) = [Sab.Cac + Sab.Sbc + Cac.Sbc]/6.
07	– – +	+ + –	P7: P(07|abc) = [Cab.Sac + Cab.Sbc + Sac.Sbc]/6.
08	– – –	+ + +	P8: P(08|abc) = [Cab.Cac + Cab.Cbc + Cac.Cbc]/6.

Table 1, based on bi-angles, is the L*R model from which all (QM) one orientation, or one-angle, two-orientations, two-outcome probabilities may be derived. *That is its purpose.* See Appendix A for all such derivations, summarized in Table 2 below.

Table 2: All the testable probabilities, in full accord with QM.

P(a+|**a**) = P(a–|**a**) = P(b+|**b**) = P(b–|**b**) = P(c+|**c**) = P(c–|**c**) =1/2. See (A4).		
See Appendix A for the following derivations, via the relevant equation (A...).		
P(ab++|ab) = Sab/2, from (A1a').	P(ac++|ac) = Sac/2, from (A2a').	P(bc++|bc) = Sbc/2, from (A3a').
P(ab+–|ab) = Cab/2, from (A1b').	P(ac+–|ac) = Cac/2, from (A2b').	P(bc+–|bc) = Cbc/2, from (A3b').
P(ab–+|ab) = Cab/2, from (A1c').	P(ac–+|ac) = Cac/2, from (A2c').	P(bc–+|bc) =Cbc/2, from (A3c').
P(ab– –|ab) = Sab/2, from (A1d').	P(ac– –|ac) = Sac/2, from (A2d').	P(bc– –|bc) = Sbc/2, from (A3d').

From Table 2: P(ab++|ab) = Sab/2, P(ac++|ac) = Sac/2, P(bc++|bc) = Sbc/2. Based on the local-realism modelled here, and in agreement with QM, there is no rational basis for constructing a Bell-inequality from these relations. That is, agreeing with QM, we have here no basis for constructing the following Bell inequality:

$$P(ab{+}{+}|ab) \leq P(ac{+}{+}|ac) + P(bc{+}{+}|bc). \qquad (?)$$

Conclusion:

Frames of reference will henceforth be useful tools in QM and its local realistic interpretation. Bell's theorem and Bell inequalities cannot be rationally constructed against the local-realism developed and analysed here. Frames of reference facilitate such analysis, the consequent L*R model here delivering every testable EPR-Bohm outcome in full accord with QM. I conclude that Bell's theorem is not applicable to the advanced (non-naive) local realism presented here. The following challenges remain: To respond to all questions; to understand any impediments and boundary conditions on modelling quantum entanglement across two classical computers.

I'd be pleased to learn of any typos or errors.

Appendix A:

This Appendix provides all the testable outcomes, all in full accord with QM. Tables A1-A3 (below) follow directly from L*R: the combination of local realism with frames of reference, making due allowance for the fact that a measurement perturbs the measured system. The consistent relations in and between the Tables are easily discerned, the physically significant results presented here following directly from the Tables and these relations. The mathematics below, after the Tables, is a continuous string, from the Tables below to Tables 1 and 2. There is no need to adjust any result or calculation to have it accord with QM.

Table A1: Reference orientation = a; ROa. Angles ab, ac; bi-angle bc = ab ± ac.			
Hidden variable class (HVC) from +/– outcomes on **a**, **b**, **c**.			Normalized HVC distribution (sum = 1) for the reference orientation **a**.
HVC ID#	HV→Alice **a b c**	HV→Bob **a b c**	
A1	+ + +	– – –	P1: P(A1|**a**) = [Cab.Cac]/2.
A2	+ + –	– – +	P2: P(A2|**a**) = [Cab.Sac]/2.
A3	+ – +	– + –	P3: P(A3|**a**) = [Sab.Cac]/2.
A4	+ – –	– + +	P4: P(A4|**a**) = [Sab.Sac]/2.
A5	– + +	+ – –	P5: P(A5|**a**) = [Sab.Sac]/2.
A6	– + –	+ – +	P6: P(A6|**a**) = [Sab.Cac]/2.
A7	– – +	+ + –	P7: P(A7|**a**) = [Cab.Sac]/2.
A8	– – –	+ + +	P8: P(A8|**a**) = [Cab.Cac]/2.

Table A1.a: Corollaries from Table A1; ROa. 2P(bc..|**a**) = average over the bi-angle bc.		
P(a+|**a**) = P(a–|**a**) = P(b+|**b**) = P(b–|**b**) = P(c+|**c**) = P(c–|**c**) =1/2.		
(A1.1a) P(ab++|**a**) = P3 + P4 = Sab/2.	(A1.2a) P(ac++|**a**) = P2 + P4 = Sac/2.	(A1.3a) P(bc++|**a**) = P2 + P6 = (Cab.Sac + Sab.Cac)/2.
(A1.1b) P(ab+–|**a**) = P1 + P2 = Cab/2.	(A1.2b) P(ac+–|**a**) = P1 + P3 = Cac/2.	(A1.3b) P(bc+–|**a**) = P1 + P5 = (Cab.Cac + Sab.Sac)/2.
(A1.1c) P(ab–+|**a**) = P7 + P8 = Cab/2.	(A1.2c) P(ac–+|**a**) = P6 + P8 = Cac/2.	(A1.3c) P(bc–+|**a**) = P4 + P8 = (Sab.Sac + Cab.Cac)/2.
(A1.1d) P(ab– –|**a**) = P5 + P6 = Sab/2.	(A1.2d) P(ac– –|**a**) = P5 + P7 = Sac/2.	(A1.3d) P(bc– –|**a**) = P3 + P7 = (Sab.Cac + Cab.Sac)/2.

Table A2: Reference orientation = b; ROb. Angles ba, bc; bi-angle ac = ba ± bc.			
Hidden variable class (HVC) from +/– outcomes on **a**, **b**, **c**.			Normalized HVC distribution (sum = 1) for the reference orientation **b**.
HVC ID#	HV→Alice **a b c**	HV→Bob **a b c**	
B1	+ + +	– – –	P1: P(B1|**b**) = [Cab.Cbc]/2.
B2	+ + –	– – +	P2: P(B2|**b**) = [Cab.Sbc]/2.
B3	+ – +	– + –	P3: P(B3|**b**) = [Sab.Sbc]/2.
B4	+ – –	– + +	P4: P(B4|**b**) = [Sab.Cbc]/2.
B5	– + +	+ – –	P5: P(B5|**b**) = [Sab.Cbc]/2.
B6	– + –	+ – +	P6: P(B6|**b**) = [Sab.Sbc]/2.
B7	– – +	+ + –	P7: P(B7|**b**) = [Cab.Sbc]/2.
B8	– – –	+ + +	P8: P(B8|**b**) = [Cab.Cbc]/2.

Table A2.b: Corollaries from Table A2; ROb. 2P(ac..|**b**) = average over the bi-angle ac.		
P(a+|**a**) = P(a–|**a**) = P(b+|**b**) = P(b–|**b**) = P(c+|**c**) = P(c–|**c**) =1/2.		
(A2.1a) P(ab++|**b**) = P3 + P4 = Sab/2.	(A2.2a) P(bc++|**b**) = P2 + P6 = Sbc/2.	(A2.3a) P(ac++|**b**) = P2 + P4 = (Cab.Sbc + Sab.Cbc)/2.
(A2.1b) P(ab+–|**b**) = P1 + P2 = Cab/2.	(A2.2b) P(bc+–|**b**) = P1 + P5 = Cbc/2.	(A2.3b) P(ac+–|**b**) = P1 + P3 = (Cab.Cbc + Sab.Sbc)/2.
(A2.1c) P(ab–+|**b**) = P7 + P8 = Cab/2.	(A2.2c) P(bc–+|**b**) = P4 + P8 = Cbc/2.	(A2.3c) P(ac–+|**b**) = P6 + P8 = (Sab.Sbc + Cab.Cbc)/2.
(A2.1d) P(ab– –|**b**) = P5 + P6 = Sab/2.	(A2.2d) P(bc– –|**b**) = P3 + P7 = Sbc/2.	(A2.3d) P(ac– –|**b**) = P5 + P7 = (Sab.Cbc + Cab.Sbc)/2.

Table A3: Reference orientation = c; ROc. Angles ca, cb; bi-angle ab = ca ± cb.			
Hidden variable class (HVC) from +/– outcomes on **a**, **b**, **c**.			Normalized HVC distribution (sum = 1) for reference orientation **c**.
HVC ID#	HV→Alice **a b c**	HV→Bob **a b c**	
C1	+ + +	– – –	P1: P(C1|**c**) = [Cac.Cbc]/2.
C2	+ + –	– – +	P2: P(C2|**c**) = [Sac.Sbc]/2.
C3	+ – +	– + –	P3: P(C3|**c**) = [Cac.Sbc]/2.
C4	+ – –	– + +	P4: P(C4|**c**) = [Sac.Cbc]/2.
C5	– + +	+ – –	P5: P(C5|**c**) = [Sac.Cbc]/2.
C6	– + –	+ – +	P6: P(C6|**c**) = [Cac.Sbc]/2.
C7	– – +	+ + –	P7: P(C7|**c**) = [Sac.Sbc]/2.
C8	– – –	+ + +	P8: P(C8|**c**) = [Cac.Cbc]/2.

Table A3.c: Corollaries from Table A3; ROc. 2P(ab..|**c**) = average over the bi-angle ab.		
P(a+|**a**) = P(a–|**a**) = P(b+|**b**) = P(b–|**b**) = P(c+|**c**) = P(c–|**c**) =1/2.		
(A3.1a) P(ac++|**c**) = P2 + P4 = Sac/2.	(A3.2a) P(bc++|**c**) = P2 + P6 = Sbc/2.	(A3.3a) P(ab++|**c**) = P3 + P4 = (Cac.Sbc + Sac.Cbc)/2.
(A3.1b) P(ac+–|**c**) = P6 + P8 = Cac/2.	(A3.2b) P(bc+–|**c**) = P4 + P8 = Cbc/2.	(A3.3b) P(ab+–|**c**) = P1 + P2 = (Cac.Cbc + Sac.Sbc)/2.
(A3.1c) P(ac–+|**c**) = P1 + P3 = Cac/2.	(A3.2c) P(bc–+|**c**) = P1 + P5 = Cbc/2.	(A3.3c) P(ab–+|**c**) = P7 + P8 = (Sac.Sbc + Cac.Cbc)/2.
(A3.1d) P(ac– –|**c**) = P5 + P7 = Sac/2.	(A3.2d) P(ac– –|**c**) = P3 + P7 = Sbc/2.	(A3.3d) P(ab– –|**c**) = P5 + P6 = (Sac.Cbc + Cac.Sbc)/2.

In an intuitive notation, defined by the following relations, we have from (A1.1a) and (A2.1a) in Tables A1 and A2 above, respectively:

$$P(ab{+}{+}|ab) = [P(ab{+}{+}|\mathbf{a}) + P(ab{+}{+}|\mathbf{b})]/2 = Sab/2. \qquad \text{(A0a)}$$

$$P(ab{+}{+}|abc) = [P(ab{+}{+}|\mathbf{a}) + P(ab{+}{+}|\mathbf{b}) + P(ab{+}{+}|\mathbf{c})]/3 = [2P(ab{+}{+}|ab) + P(ab{+}{+}|\mathbf{c})]/3. \qquad \text{(A0b)}$$

$$\therefore\ P(ab{+}{+}|ab) = [3P(ab{+}{+}|abc) - P(ab{+}{+}|\mathbf{c})]/2 = Sab/2. \qquad \text{(A0c)}$$

Note that P(ab++|ab) is independent of ROc, as expected from our definition of L*R. P(ab++|**c**) is given at (A3.3a) above. Also, from (A1.1b) and (A2.1b) in Tables A1 and A2 above, respectively:

$$P(ab{+}{-}|ab) = [P(ab{+}{-}|\mathbf{a}) + P(ab{+}{-}|\mathbf{b})]/2 = Cab/2. \qquad \text{(A0d)}$$

$$P(ab{+}{-}|abc) = [P(ab{+}{-}|\mathbf{a}) + P(ab{+}{-}|\mathbf{b}) + P(ab{+}{-}|\mathbf{c})]/3 = [2P(ab{+}{-}|ab) + P(ab{+}{-}|\mathbf{c})]/3. \qquad \text{(A0e)}$$

$$\therefore\ P(ab{+}{-}|ab) = [3P(ab{+}{-}|abc) - P(ab{+}{-}|\mathbf{c})]/2 = Cab/2. \qquad \text{(A0f)}$$

P(ab+–|ab) is independent of ROc, as required; and so on, for any combination of variables. P(ab+–|**c**) is given at (A3.3b) above. Thus Table 1 provides the correct *testable* outcomes; confirming the *untestable* two-angle outcomes provided by Tables A1, A2, A3.

From (A0c) and (A0f) we derive the general formula:

$$\mathbf{P(xy{\cdot\cdot}|xy) = [3P(xy{\cdot\cdot}|xyz) - P(xy{\cdot\cdot}|z)]/2 = Fxy/2}, \qquad \text{(A0g)}$$

where x, y, z represent a, b, c in any order, and (in accord with our notation) the function F is S if ·· is + + or – –; C if ·· is + –, – +. The testable results for any *orientation* being clear from the above Tables, see (A4) below, all the testable results for any *angle* now follow:

$$P(ab{+}{+}|abc) = P(03|abc) + P(04|abc) = [Sab.Cac + Sab.Sbc + Cac.Sbc + Sab.Sac + Sab.Cbc + Sac.Cbc]/6 = [2Sab + Cac.Sbc + Sac.Cbc]/6 = [Sab + P(ab{+}{+}|\mathbf{c})]/3. \qquad \text{(A1a).}$$

$$\text{From (A0g): } P(ab{+}{+}|ab) = [3Pab({+}{+}|abc) - P(ab{+}{+}|\mathbf{c})]/2 = Sab/2. \qquad \text{(A1a')}$$

$$P(ab{+}{-}|abc) = P(01|abc) + P(02|abc) = [Cab.Cac + Cab.Cbc + Cac.Cbc + Cab.Sac + Cab.Sbc + Sac.Sbc]/6 = [2Cab + Cac.Cbc + Sac.Sbc]/6 = [Cab + P(ab{+}{-}|\mathbf{c})]/3. \qquad \text{(A1b)}$$

$$\text{From (A0g): } P(ab{+}{-}|ab) = [3Pab({+}{-}|abc) - P(ab{+}{-}|\mathbf{c})]/2 = Cab/2. \qquad \text{(A1b')}$$

P(ab–+|abc) = P(07|abc) + P(08|abc) = [Cab.Sac + Cab.Sbc + Sac.Sbc + Cab.Cac + Cab.Cbc + Cac.Cbc]/6 = [2Cab + Sac.Sbc + Cac.Cbc]/6 = [Cab + P(ab–+|**c**)]/3. (A1c)

From (A0g): P(ab–+|ab) = [3Pab(–+|abc) – P(ab–+|**c**)]/2 = Cab/2. (A1c')

P(ab– –|abc) = P(05|abc) + P(06|abc) = [Sab.Sac + Sab.Cbc + Sac.Cbc + Sab.Cac + Sab.Sbc + Cac.Sbc]/6 = [2Sab + Sac.Cbc + Cac.Sbc] = [Sab + P(ab– –|**c**)]/3. (A1d)

From (A0g): P(ab– –|ab) = [3Pab(– –|abc) – P(ab– –|**c**)]/2 = Sab/2. (A1d')

P(ac++|abc) = P(02|abc) + P(04|abc) = [Cab.Sac + Cab.Sbc + Sac.Sbc + Sab.Sac + Sab.Cbc + Sac.Cbc]/6 = [2Sac + Cab.Sbc + Sab.Cbc]/6 = [Sac + P(ac++|**b**)]/3. (A2a)

From (A0g): P(ac++|ac) = [3Pac(++|abc) – P(ac++|**b**)]/2 = Sac/2. (A2a')

P(ac+–|abc) = P(01|abc) + P03|abc) = [Cab.Cac + Cab.Cbc + Cac.Cbc + Sab.Cac + Sab.Sbc + Cac.Sbc]/6 = [2Cac + Cab.Cbc + Sab.Sbc]/6 = [Cac + P(ac+–|**b**)]/3. (A2b)

From (A0g): P(ac+–|ac) = [3Pac(+–|abc) – P(ac+–|**b**)]/2 = Cac/2. (A2b')

P(ac–+|abc) = P(06|abc) + P(08|abc) = [Sab.Cac + Sab.Sbc + Cac.Sbc + Cab.Cac + Cab.Cbc + Cac.Cbc]/6 = [2Cac + Sab.Sbc + Cab.Cbc]/6 = [Cac + P(ac–+|**b**)]/3. (A2c)

From (A0g): P(ac–+|ac) = [3Pac(–+|abc) – P(ac–+|**b**)]/2 = Cac/2. (A2c')

P(ac– –|abc) = P(05|abc) + P(07|abc) = [Sab.Sac + Sab.Cbc + Sac.Cbc + Cab.Sac + Cab.Sbc + Sac.Sbc]/6 = [2Sac + Sab.Cbc + Cab.Sbc] = [Sac + P(ac– –|**b**)]/3. (A2d)

From (A0g): P(ac– –|ac) = [3Pac(– –|abc) – P(ac– –|**b**)]/2 = Sac/2. (A2d')

P(bc++|abc) = P(02|abc) + P(06|abc) = [Cab.Sac + Cab.Sbc + Sac.Sbc + Sab.Cac + Sab.Sbc + Cac.Sbc]/6 = [2Sbc + Cab.Sac + Sab.Cac]/6 = [Sbc + P(bc++|**a**)|]/ (A3a)

From (A0g): P(bc++|bc) = [3Pbc(++|abc) – P(bc++|**a**)]/2 = Sbc/2. (A3a')

P(bc+–|abc) = P(01|abc) + P(05|abc) = [Cab.Cac + Cab.Cbc + Cac.Cbc + Sab.Sac + Sab.Cbc + Sac.Cbc]/6 = [2Cbc + Cab.Cac + Sab.Sac]/6 = [Cbc + P(bc+–|**a**)]/3. (A3b)

From (A0g): P(bc+–|bc) = [3Pbc(+–|abc) – P(bc+–|**a**)]/2 = Cbc/2. (3b')

P(bc–+|abc) = P(04|abc) + P(08|abc) = [Sab.Sac + Sab.Cbc + Sac.Cbc + Cab.Cac + Cab.Cbc + Cac.Cbc]/6 = [2Cbc + Sab.Sac + Cab.Cac]/6 = [Cbc + P(bc–+|**a**)]/3. (A3c)

From (A0g): P(bc–+|bc) = = [3Pbc(–+|abc) – P(bc–+|**a**)]/2 = Cbc/2 (A3c')

Pbc(– –|abc) = P(03|abc) + P(07|abc) = [Sab.Cac + Sab.Sbc + Cac.Sbc + Cab.Sac + Cab.Sbc + Sac.Sbc]/6 = [2Sbc + Sab.Cac + Cab.Sac] = [Sbc + P(bc– –|**a**)]/3. (A3d)

From (A0g): P(bc– –|bc) = [3Pbc(– –|abc) – P(bc– –|**a**)]/2 = Sbc/2. (A3d')

To complete our demonstration of L*R's accord with QM, we now show that the test results applicable to any orientation are also those of QM. We give one example from Table 1; other results follow similarly.

$$\begin{aligned} P(a+|\mathbf{a}) &= P(01|abc) + P(02|abc) + P(03|abc) + P(04|abc) \\ &= [Cab.Cac + Cab.Cbc + Cac.Cbc + Cab.Sac + Cab.Sbc + Sac.Sbc \\ &\quad + Sab.Cac + Sab.Sbc + Cac.Sbc + Sab.Sac + Sab.Cbc + Sac.Cbc]/6 \\ &= [2Cab + 2Sab + Cbc + Sbc]/6 = 1/2. \end{aligned} \tag{A4}$$

Acknowledgments: Forthcoming.

References:

[1] http://www.physicsforums.com/member.php?u=74950
[2] Sources to be added later.
[3] http://en.wikipedia.org/wiki/Sakurai%27s_Bell_inequality

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