Assume you drop a ball

Begin with this (x positive down, gravity positive down)

$$\ddot{x} = g$$

Integrate (ignore initial conditions)

$$\dot{x} = gt$$

Integrate (ignore initial conditions)

$$x = \frac{1}{2} g t^2$$

Solve for time

$$\sqrt{\frac{1}{g} 2x} = t$$

Express velocity as a function of time

$$\dot{x} = g\sqrt{\frac{1}{g} 2x}$$

Manipulate, the above to get

$$\dot{x}^2 = 2gx$$

State the KE

$$T = \frac{1}{2} m \dot{x}^2$$

Or, using the pink equation, we also COULD get (this could be where I go wrong)

$$T = \frac{1}{2} m 2gx = mgx$$

Take derivative with respect to velocity of each form of T

$$\left(\frac{\partial T}{\partial \dot{x}_i}\right) = m\dot{x} \qquad \left(\frac{\partial T}{\partial \dot{x}_i}\right) = 0$$

Take the time derivative of that

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) = m\ddot{x} \qquad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) = 0$$

Take the derivative with position

$$\frac{\partial T}{\partial x} = 0 \qquad \frac{\partial T}{\partial x} = mg$$

Turn to the Euler Lagrange and fill in each color

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)-\frac{\partial T}{\partial x_i}=-\frac{\partial V}{\partial x_i}$$

$$m\ddot{x}=-\frac{\partial V}{\partial q_i} \qquad\qquad -mg=-\frac{\partial V}{\partial q_i}$$

Equate the blue terms and recover this

$$\ddot{x}=-g$$

But that is a contradiction.