Feynman Diagrams In String Theory

Edward Witten

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I will aim to explain the minimum about string perturbation theory that every quantum physicist should know.

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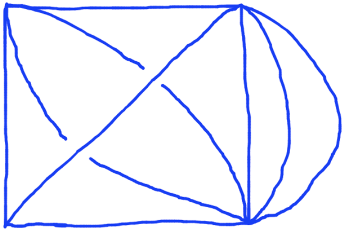

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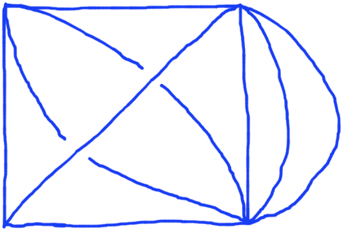

we assign a propagator $1/(p^2+m^2)$ to each line. We can write

$$\frac{1}{p^2+m^2} = \int_0^\infty \mathrm{d}t \exp(-t(p^2+m^2)),$$

where t is the Schwinger parameter or proper time.

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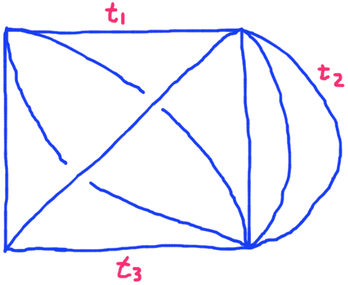

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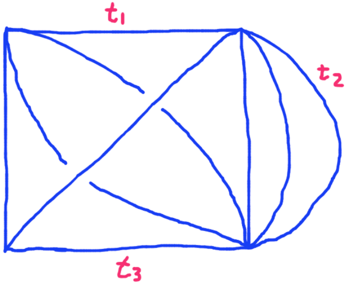

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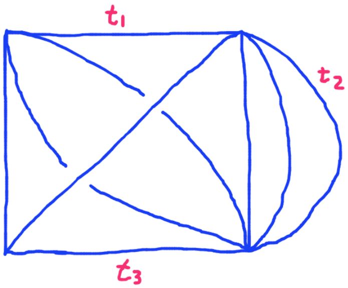

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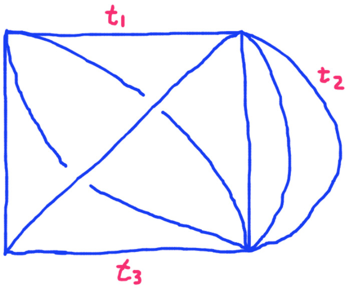

It is convenient to think of the Feynman graph Γ as a singular 1-manifold, with singularities at the vertices. What it means to assign the length parameters t_i to the graph is just that we have a Riemannian metric on Γ. Up to diffeomorphisms of the internal lines in Γ, the only invariants of such a metric are the lengths of the line segments, that is, the t_i.

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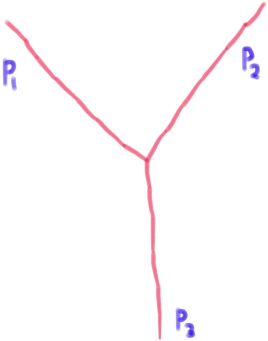


we want a delta function

$$(2\pi)^4\delta^4(p_1 + p_2 + p_3).$$

We can conveniently get that delta function from

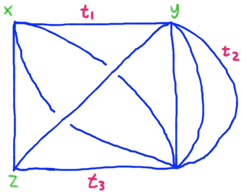
$$\int \mathrm{d}^4 x \exp(i \sum_i p_i \cdot x) = (2\pi)^4 \delta^4(\sum_i p_i).$$

So we assign a spatial coordinate to each vertex, and we write the propagator in position space as

$$G(x,y) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{e^{ip\cdot(x-y)}}{p^2+m^2}$$

$$= \int_0^\infty \mathrm{d}t \frac{\mathrm{d}^4 p}{(2\pi)^4} \exp(ip\cdot x - t(p^2+m^2)).$$

So now we have a slightly new way to interpret a Feynman diagram:



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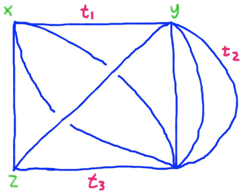

We integrate over a position parameter x for each vertex, and a length parameter t for each line. In addition, each line has a factor

$$G(x, y; t) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \; e^{ip\cdot(x-y)-t(p^2+m^2)}.$$

However, in addition to inventing Feynman diagrams, Feynman also taught us how to interpret the function

$$G(x,y;t) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\ e^{ip\cdot(x-y)-t(p^2+m^2)}.$$

We think about a non relativistic point particle with Hamiltonian

$$H = p^2 + m^2, \qquad\qquad p = -id/dx.$$

The action for such a particle is

$$I = \int \mathrm{d}t \left(\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + m^2\right),$$

which is just the action for a non relativistic point particle with $2m = 1$, and a constant m^2 added to the Lagrangian density.

According to Feynman, $G(x, y; t)$ can be obtained as an integral over all paths $X(t')$ for which $X(0) = y$, $X(t) = x$, or in other words all paths by which the particle travels from y to x in time t:

$$G(x, y; t) = \int DX(t') \exp\left(-\int_0^t \mathrm{d}t' \left(\sum_i \left(\frac{\mathrm{d}X^i}{\mathrm{d}t'}\right)^2 + m^2\right)\right).$$

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This is the basic Feynman path integral of non-relativistic quantum mechanics, which you can read about in for example the book by Feynman and Hibbs.

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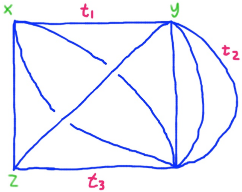

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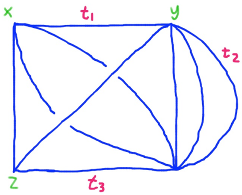

we integrate over all possible Riemannian metrics on the graph, which amounts to integrating over all length parameters $t_1, t_2, \ldots$. We also integrate over all possible locations of the vertices x, y, z in space-time, and also over all possible maps of the lines connecting the vertices into space-time.

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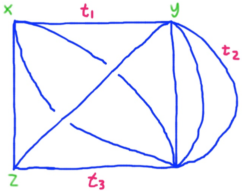

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$$I = \int_\Gamma \mathrm{d}s \sqrt{h} \left(h^{-1} \sum_i g_{ij} \frac{\mathrm{d}X^i}{\mathrm{d}s} \frac{\mathrm{d}X^j}{\mathrm{d}s} + m^2 \right).$$

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An important point is that the integral over each Schwinger parameter t has two ends. There is $t \to \infty$ which generates the pole of the propagator:

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We do not want to do without this region since the physical interpretation of quantum field theory depends crucially on the pole of the propagator!

The other end of the t integral is responsible for the fact that the propagator is singular at short distances:

$$\int_0^\Lambda \mathrm{d}t \int \frac{\mathrm{d}^d p}{(2\pi)^d} e^{ip\cdot(x-y)-t(p^2+m^2)} \sim \frac{1}{|x-y|^{d-2}}.$$

This singular short distance behavior of the propagator comes completely from the small t part of the integral.

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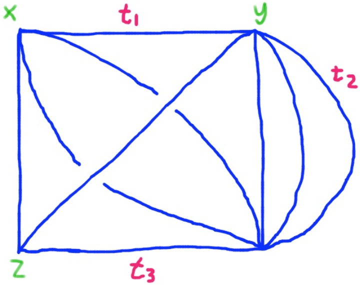

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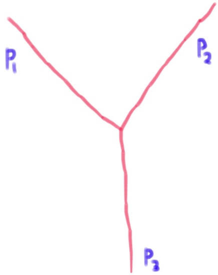

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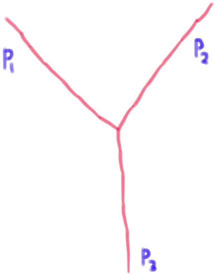

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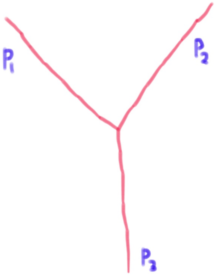

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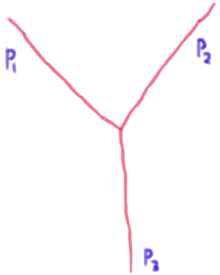

should be labeled by which types of particles are attached to it. A vertex has a coupling constant, which may depend on which particles are chosen. The Feynman rules may tell us to place at a vertex not just a coupling "constant" but a more general factor that may depend on the momenta or possibly the polarizations of the particles.

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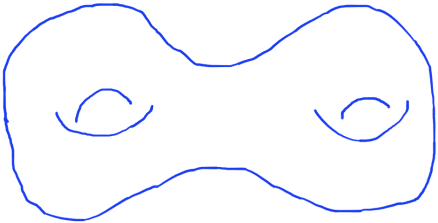

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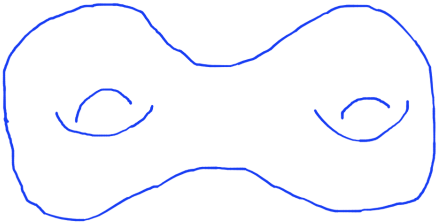

. I will call such a 2-manifold "the string worldsheet."

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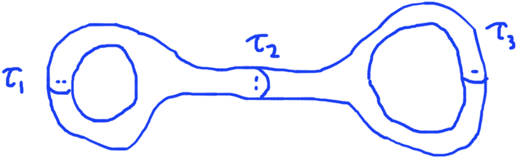
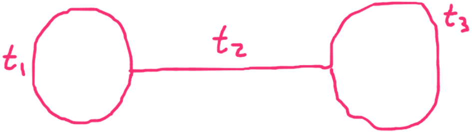
$$h_{ij} \cong h_{ij} e^{2\sigma}$$

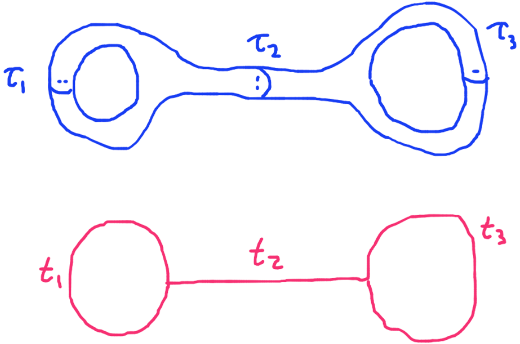
for any function σ.

A 19th century result says that, up to diffeomorphisms and conformal or Weyl transformations, a two-manifold Σ only depends on finitely many parameters:





The moduli τ_i of the Riemann surface are rather analogous to the proper times t_i of the edges of a corresponding Feynman graph, except that they are complex while the t_i are real.



More concretely,

$$\tau_i = s_i + it_i$$

where t_i is the "length" of a tube and s_i is a "twist angle."

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$$I = \int_\Sigma d^2\sigma \sqrt{\det g} \left(g^{\alpha\beta} \partial_\alpha X^I \partial_\beta X^J G_{IJ}(X) \right).$$

The X's describe a map from the two-manifold Σ to a spacetime M, which can have D dimensions and which I've endowed with a metric tensor $G_{IJ}(X)$.

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$$\int_\Sigma \mathrm{d}^2\sigma \sqrt{\det h}\, R$$

is a total derivative (topological invariant). So just as in ordinary quantum field theory, we basically have to only consider the action for the matter fields.

To develop the theory, we are supposed to (i) for a fixed Σ, do a Feynman path integral over the fields $X = (X^1, \ldots, X^D)$, and (ii) then integrate over the moduli $\tau_1, \tau_2, \ldots$ and sum over all topological choices for Σ. The last step is the analog of summing over all Feynman graphs in ordinary field theory.

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This means that the 1-loop contribution to the effective action is

$$I^* = \frac{1}{2}\mathrm{Tr}\,\log(-\nabla^2 + m^2) = \frac{1}{2}\int_0^\infty \frac{\mathrm{d}t}{t}\exp(-tH)$$

where $H = p^2 + m^2 = -\nabla^2 + m^2$.

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It is very instructive in field theory to see that the factor $1/2t$ comes from the symmetries of the graph.

This integral diverges for $t \to 0$. What we are about to see is that the string theory problem is similar, except that the integral only goes over $t \geq 1$, so there will be no divergence.

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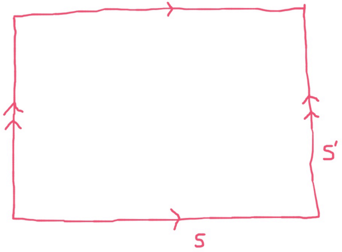
(I will take a shortcut and assume the torus is conformally equivalent to a rectangle with opposite sides glued together rather than a more general parallelogram. This doesn't affect the conclusion, but it shortens the explanation.)

In field theory, the circle has a circumference t. In string theory, a rectangle with a flat metric would have two parameters s and s':



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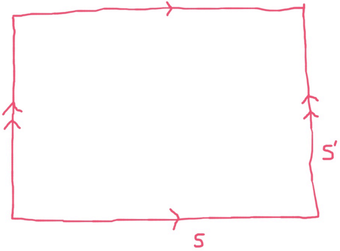

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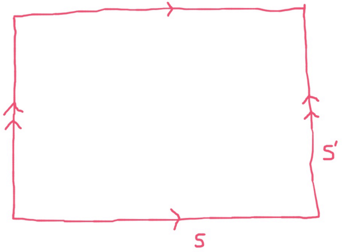

We could view the rectangle as describing a string of circumference s propagating through a proper time s' or a string of circumference s' propagating through a proper time s. Either picture is correct. In any event, in string theory, because of conformal invariance, only the ratio s/s' (or its inverse s'/s) is meaningful.

The string theory formula will reduce approximately to field theory if, say, $s >> s'$. Then we can think in terms of a string of circumference s' propagating for a proper time s. Because of conformal invariance we can set $s' = 1$ and identify s with the proper time t of a field theory: $t = s/s' = s$. So the integral that in field theory is an integral over the proper time t is in string theory replaced by an integral over the ratio s/s'.

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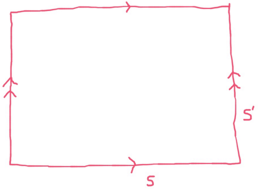

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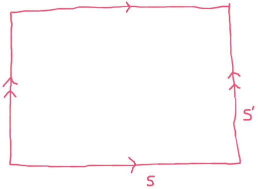

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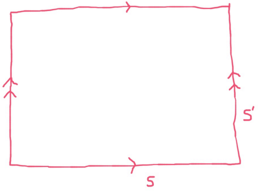

Because of this symmetry, we are free to restrict the integration region to $s \geq s'$. In other words, if we set $t = s/s'$, we are restricted to $t \geq 1$.

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What is true but much less trivial is the following statement: The infrared behavior of string theory matches the infrared behavior of a field theory with appropriate light particles and interactions.

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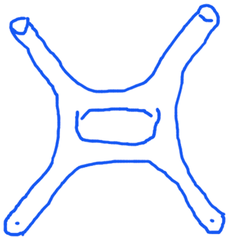

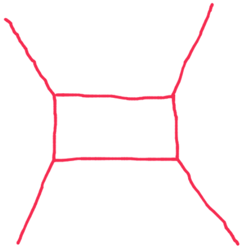

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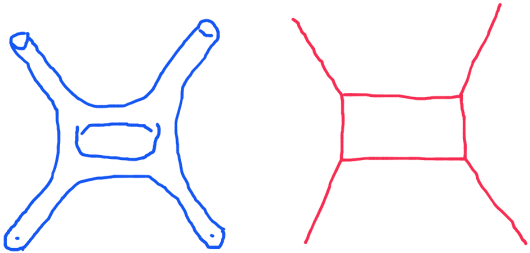

In other words, the fields are the vibrational states of a string.

If we change the metric G_{IJ} of M a little bit, by $G \to G + \delta G$, the action changes by

$$\delta I = \int_\Sigma d^2\sigma \sqrt{\det h} \left(h^{\alpha\beta} \partial_\alpha X^I \partial_\beta X^J \delta G_{IJ}(X) \right).$$

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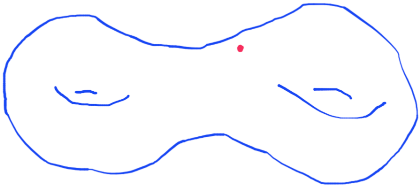
This causes an operator insertion on the Riemann surface somewhere (we have to integrate over where):



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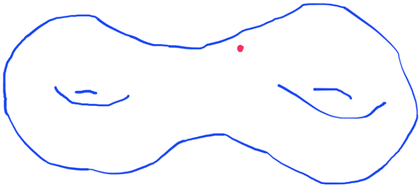
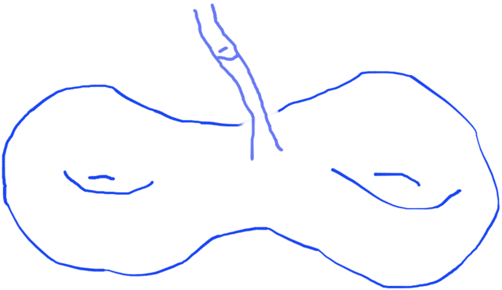


For the case that the change in the action comes from a change in the metric in spacetime, the operator is $\mathcal{O} = h^{\alpha\beta} \partial_\alpha X^I \partial_\beta X^J \delta G_{IJ}(X)$.

By the magic of conformal invariance, the point where we inserted the operator can be projected to infinity:



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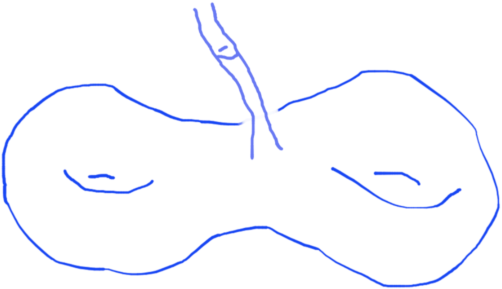

So it corresponds to one of the external fields – and if we shift the expectation value of that field, this amounts to shifting the metric G_{IJ} in spacetime.

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So the dynamics of gravity in spacetime is part of what string theory describes. A shift in the metric of spacetime is a shift in the expectation value of one of the string fields; or differently put, one of the string fields is the spacetime metric.

Going down this road (and incorporating spacetime supersymmetry to avoid some infrared problems), one arrives at a systematic way to calculate quantum processes involving gravitons, free of the ultraviolet divergences that one gets if one tries to quantize Einstein's theory directly.

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