5 Magnetic Fields

5.1 Introduction

In the previous sections we have seen how electric charges give rise to forces on other charges and to electric fields. In this section we will see that where electric charges are in motion, they give rise to *magnetic* fields.

We'll look at how these fields are defined, and can be represented by magnetic field lines, by considering the case of permanent magnets. We'll also examine magnetic fields produced around current-carrying conductors (the basis of electromagnets).

Next we will look at the forces on current-carrying conductors and moving charges in magnetic fields and the torque on a current-carrying coil in a magnetic field (the basis of electric motors). Finally, we'll consider the force between two parallel conductors (from which we get the definition of the ampere).

5.2 Permanent magnets

The magnetic field around a permanent magnet originates from the motion of the electrons within the atoms of the magnetic material. Permanent magnets have equal and opposite *poles*, generally referred to simply as *north* and *south* poles. And it is well known, from common experience, that ***opposite* magnetic poles *attract***, while ***like* poles *repel***.

It is worth noting briefly the origins of the definitions of the magnetic poles. A freely suspended bar magnet will align itself with the earth's magnetic field[39] such that one end – and always the same end – points towards the geographical north and the other end points south. Hence, the terms *north-seeking pole* and *south-seeking pole* were adopted to identify the ends of the magnet (but now these are usually shortened to just *north* and *south*). Clearly the *north*-seeking pole of the magnet must be attracted by the earth's magnetic pole located near the *geographical* north pole – and, equally clearly, this means that the earth's magnetic pole in the north must, by definition, in *magnetic* terms, be a *south* pole! Conversely, the magnetic pole located at the south of the globe is, magnetically speaking, a *north* pole (see Figure 5.1).

Magnetic fields, like electric fields, can be represented by field lines whose concentration indicates the strength of the field. The direction of a magnetic field at a point is taken as the

[39] The earth has an electric field resembling that of a bar magnet. The origins of the earth's magnetic field are not fully understood but are thought to result from electric currents circulating in its molten core.

direction of the force experienced by a *north* pole placed at that point. So the field lines come *out* of a north pole and *into* a south pole (see Figure 5.1).

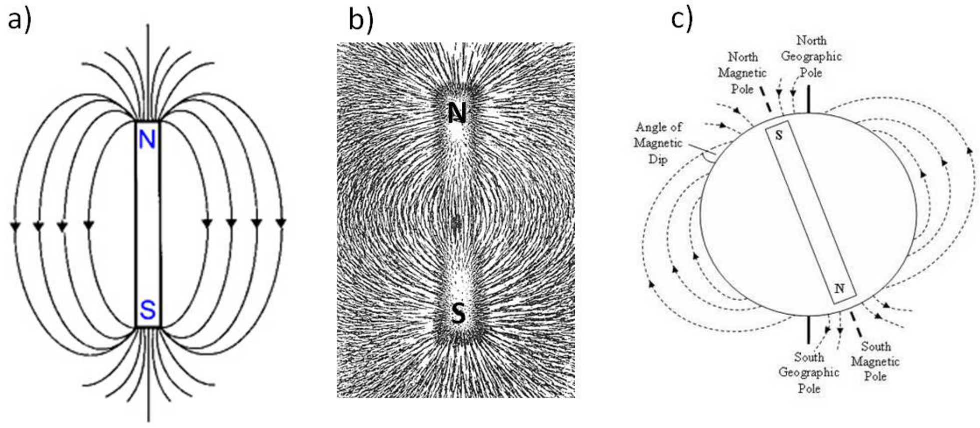


Figure 5.1 Magnetic field lines a) for a simple bar magnet b) as demonstrated using iron filings and c) the magnetic field of the earth, showing the field lines and designation of the geographic and magnetic poles.

5.3 Magnetic fields around conductors.

In the introduction it was stated that moving charges give rise to magnetic fields. Since electric current is a flow of charge, as expected, we find magnetic fields associated with current-carrying conductors such as wires.

Here we will briefly consider the nature of the magnetic fields that result from current flowing through a long, straight conductor (e.g. a wire), a flat circular coil and a solenoid (i.e. a long cylindrical coil).

In the following discussions we will often use the common notation for showing vectors (fields, current, force etc) that are perpendicular to the page – in other words coming out of, or going into, the page. So, the diagram below reminds us of this notation.

Figure 5.2 Notation for vectors at right angles to the page

Figure 5.3(a) shows the magnetic field pattern around a long straight conductor running at right angles to the page, carrying a current that is flowing *into* the page. The field consists of a series of concentric circles around the conductor[40], the direction of the field being *clockwise* (as shown by the arrows). Figure 5.3(b) shows the magnetic field for a similar conductor when the current is flowing *out* of the page. The direction of the field is now *anticlockwise*.

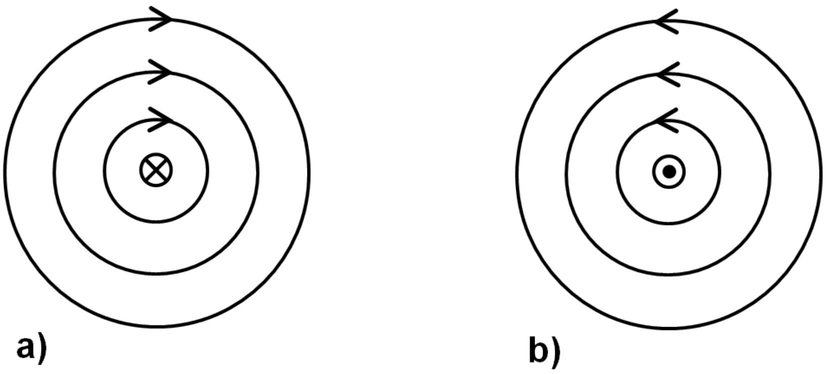


Figure 5.3 Magnetic field around long, straight conductors carrying a current (a) into the page and (b) out of the page.

The strength of the magnetic field around long straight conductors (in air or a vacuum) is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

where ***B*** **is the magnetic field strength** (also referred to as the *magnetic flux density*) and has units of tesla[41] (T). $\boldsymbol{\mu_0}$ **is the *permeability* of free space**[42] and is equal to $4\pi \times 10^{-7}$ TmA^{-1}. *r* is the distance from the centre of the conductor.

There is a *right-hand rule* for determining the direction of a magnetic field around a current-carrying conductor. Imagine grasping the conductor with your right hand, with the thumb pointing in the direction that the (conventional) current is flowing. The direction of the field is then given by the direction that the fingers are pointing around the conductor – as shown in Figure 5.4.

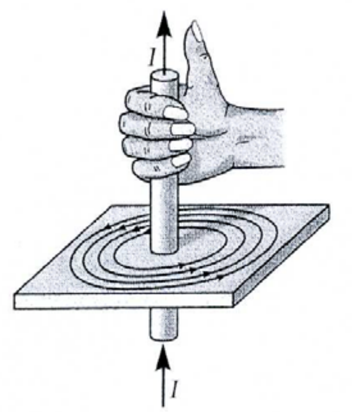


Figure 5.4 The right-hand rule for determining the direction of a magnetic field around a current-carrying conductor

[40] In this and the following examples, we are assuming that the magnetic fields are large enough that any interference from the Earth's magnetic field will be insignificant.
[41] An alternative unit for *B* is the weber per square metre (Wbm^{-2}) where 1T = 1 Wbm^{-2}.
[42] The permeability of air can be taken as equal to that of a vacuum i.e. the permeability of free space. Permeability is discussed more fully in a later section (under The Magnetic Behaviour of Materials).

The next case that we will consider is that of the magnetic field in the vicinity of a flat, circular coil – shown in the inset to Figure 5.5. The main diagram in Figure 5.5 shows a schematic representation of such a coil, viewed from above, with the plane of the coil positioned at right angles to the plane of the paper. The cross and the dot indicate the direction of the current through the opposite sides of the coil.

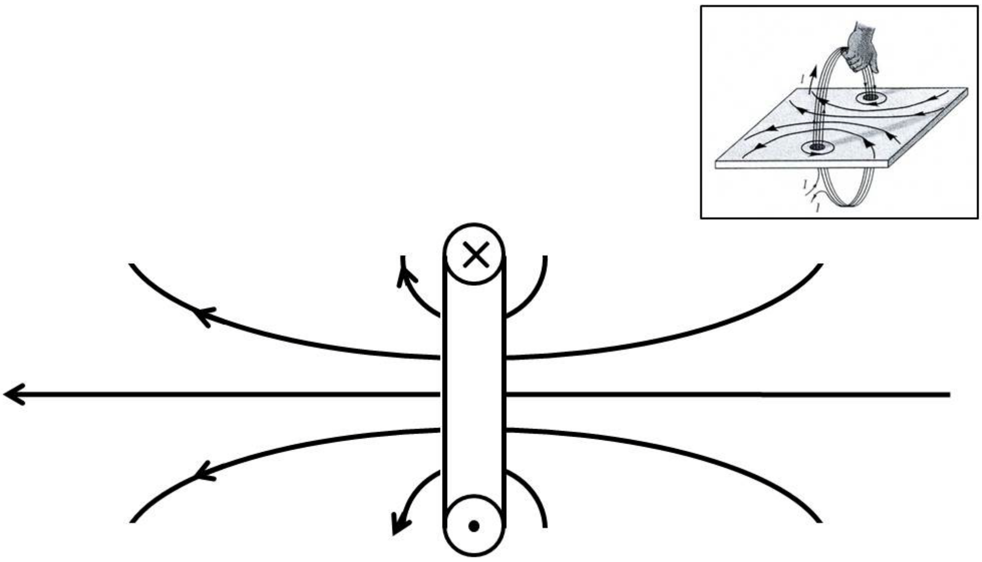


Figure 5.5 Magnetic field caused by a flat, circular, current-carrying coil

Very close to the wire the field pattern takes the form of almost concentric circles which become progressively more distorted further away as the field due to current in other parts of the coil becomes more significant. The direction of the field lines is still consistent with the right-hand rule.

The field strength *at the centre* of a flat coil is given by

$$B = \frac{N\mu_0 I}{2a}$$

where N is the number of loops in the coil and a is the radius of the loops.

The final case that we will consider is that of the magnetic field generated by a solenoid. Figure 5.6 shows the field associated with a solenoid. An interesting point to note is that there is a region of uniform magnetic field inside the solenoid (although it becomes non-uniform near the ends). Also, note that the nature of the field is such that the coil, in effect, has a north pole and a south pole – like a bar magnet.

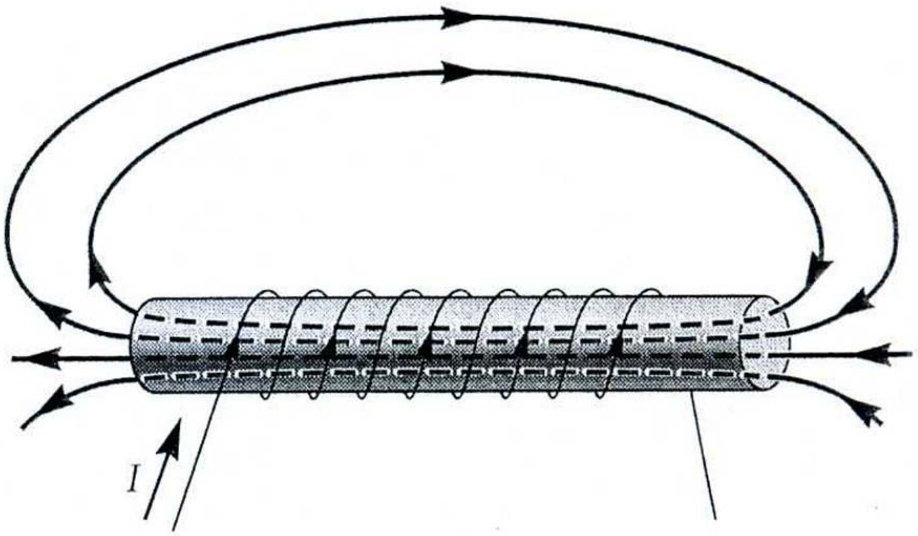


Figure 5.6 The magnetic field associated with a solenoid

The magnetic field strength *inside* the solenoid is given by the following expression, in which *n* is the number of loops per metre

$$B = n\mu_0 I$$

5.4 Force on a current-carrying conductor in a magnetic field

When a current flows through a wire that is at some angle to a magnetic field (other than parallel to the field) the wire experiences a force whose direction is perpendicular to the plane containing the current and field directions. This effect is of huge technological importance since it forms the basis of electric motors, in which electrical energy is converted to mechanical motion.

There is a second *right-hand rule* for easily identifying the direction of the force experienced by a current in a magnetic field. It is illustrated in Figure 5.7, below.

The right hand is held with the fingers extended and pointing in the direction of the magnetic field. The thumb points in the direction of the (conventional) current. The direction of the force is then given by the direction in which the palm of the hand is facing (ready to push).

As we shall see shortly, the current (thumb) doesn't have to be exactly at right angles to the field (fingers) for a force to be present – although the force is a maximum when they are perpendicular (and equal to zero if they are parallel).

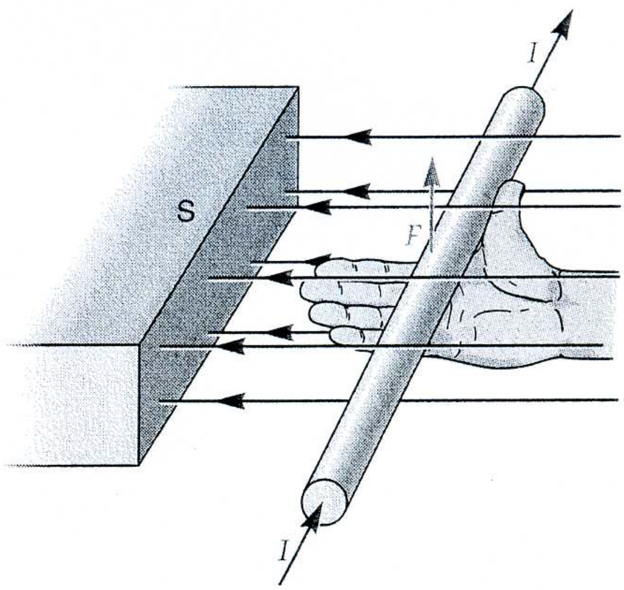


Figure 5.7 The right-hand rule for determining the direction of the force experienced by a current-carrying conductor in a magnetic field

The magnitude of the force acting on a conductor is proportional to the strength of the magnetic field expressed in terms of its magnetic flux density (*B*). It is also proportional to the size of the current (*I*) and the length of the conductor in the field (*L*).

In the case where the current direction is perpendicular to the field direction the magnitude of the force is given by

$$F = BIL \tag{5.1}$$

However, the size of this force does depend on the angle between the current and field directions – as illustrated in Figure 5.8. Where the current and field are *not* at right angles, the key parameter in determining the size of the force is the *component* of the current direction that *is* at a right angle to the field.

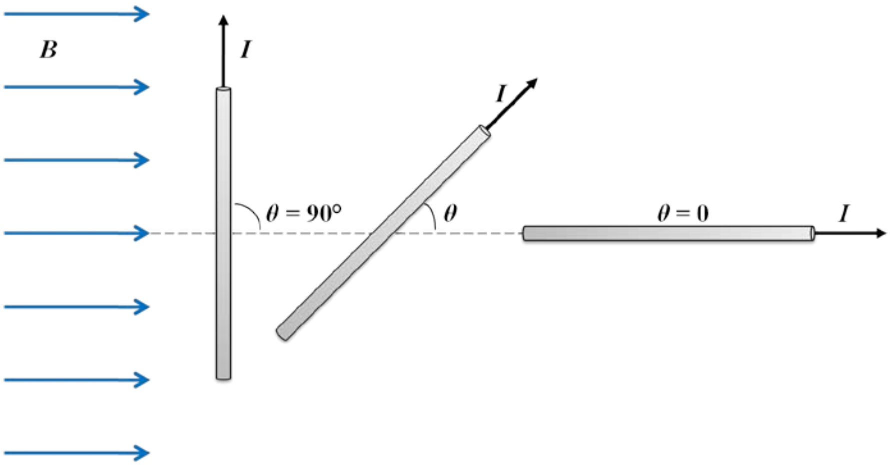


Figure 5.8 The magnitude of magnetic force depends on the angle between the current and magnetic field directions

As we have already stated, the force is a maximum (and equal to *BIL*) when the angle between the field and the current is 90°. And when the angle is 0° (in other words the current and field are parallel) the force is zero – because there is *no* component of the current that is perpendicular to the field.

It is clear from Figure 5.8 that the component of the current that is perpendicular to the field is simply given by $I\sin\theta$, which means that the force acting on that component will be proportional to $I\sin\theta$ and equation (5.1) can be re-written as

$$\boxed{F = BIL\sin\theta} \tag{5.2}$$

Example: A straight wire 100 mm long experiences a force of 15×10^{-3} N while it carries a current of 3 amps perpendicular to a uniform magnetic field.

a) Find the flux density of the field

b) Find the magnitude of the force acting on the wire if it makes an angle of 60° with the field.

a) Rearranging equation (5.2) gives

$$\begin{aligned} B &= \frac{F}{IL\sin\theta} = \frac{15\times10^{-3}}{3\times0.1\times\sin 90} \\ &= 0.05\ \mathrm{T} \end{aligned}$$

b) If the wire makes an angle of 60° with the field

$$\begin{aligned} F &= BIL\sin\theta \\ &= 0.05\times3\times0.1\times\sin 60 \\ &= 13\times10^{-3}\ \mathrm{N} \end{aligned}$$

5.5 The force on a moving charge in a magnetic field

We have looked at the force experienced by a current – a flow of charges – in a magnetic field. Now we will consider the force felt by a single charge moving in a magnetic field.

Equation (5.2) can easily be adapted to obtain the force acting on a single electron in a current-carrying conductor.

We use equation (3.2) for the drift velocity of electrons in a conductor, which can be arranged in the following form

$$I = nAve$$

remembering that n is the number of free electrons per unit volume, A is the cross sectional area of the conductor, v is the drift velocity and e is the electronic charge.

We can substitute this expression for I into our equation for the force, which gives

$$F_{current} = BnAveL\sin\theta$$

where F_{current} is the *total* force on all the charges making up the current.

Since AL (area × length) is equal to volume, and n is the number of electrons per unit volume, the number of free electrons in a length L of wire is nAL. So, if we want to know the force F acting on just one electron we just divide this expression by nAL, which gives

$$F = \frac{F_{\text{current}}}{\text{nAL}} = \frac{BnAveL\sin\theta}{nAL} = Bve\sin\theta$$

Since we can consider any charge moving as representing a part of a current, we can use this expression to say that, in general, a charge q moving with a speed v at an angle θ relative to the direction of a magnetic field B will experience a force, given by

$$\boxed{F = qvB\sin\theta} \qquad (5.3)$$

If the charge is moving perpendicularly to the field, then $F = qvB$ because $\sin 90° = 1$.

From this it follows that a magnetic field will deflect a beam of charged particles passing through it. The direction of the deflecting force is once again found using the right-hand rule (remembering that a beam of *negative* particles are considered to be travelling in the *opposite direction* to the direction of the conventional current – so the thumb points in the *opposite* direction to the direction of motion of a negatively charged particle).

Because the force acting on the charged particle is always acting at right angles to its direction of motion, the force cannot change the speed of the particle – in other words *the force does no work on the moving charge*. It does, however, change its direction and, assuming the charge is moving through a uniform magnetic field, the path that it follows will be a circular one, as shown in Figure 5.9.

If we recall that centripetal force is simply given by mv^2/r, we can derive an equation describing the path of a charged particle moving at right angles to a uniform magnetic field in terms of its velocity (v) and mass (m).

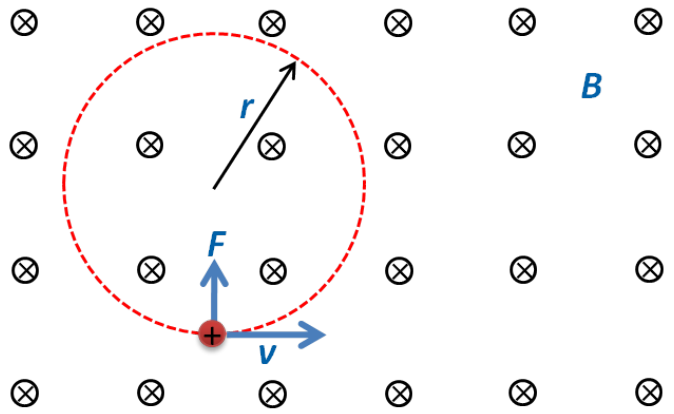


Figure 5.9 Motion of a charged particle in a uniform magnetic field

We can consider that, for a charge moving in a circular path at right angles to a magnetic field, the *centripetal* force (that is stopping it 'flying off' at a tangent) is being provided by the *magnetic* force on the charge. So, we can equate these two forces and write

$$\boxed{\frac{mv^2}{r} = qvB} \tag{5.4}$$

This equation, which tells us that the radius of the path of a charged particle moving in a magnetic field is proportional to the mass of the particle, is the basis of an instrument called a mass spectrometer.

Mass spectrometers effectively separate and identify particles by their masses. Their masses are determined by measuring the radius of curvature of the particles as they pass through a uniform, perpendicular magnetic field.

Mass spectrometers are used in numerous applications – for example, security scanning, gas analysis, analytical chemistry etc, for their ability to identify unknown substances.

The substance is first ionised and the ions are 'injected' into the mass spectrometer at a known velocity. Their path can be calculated from recording their position

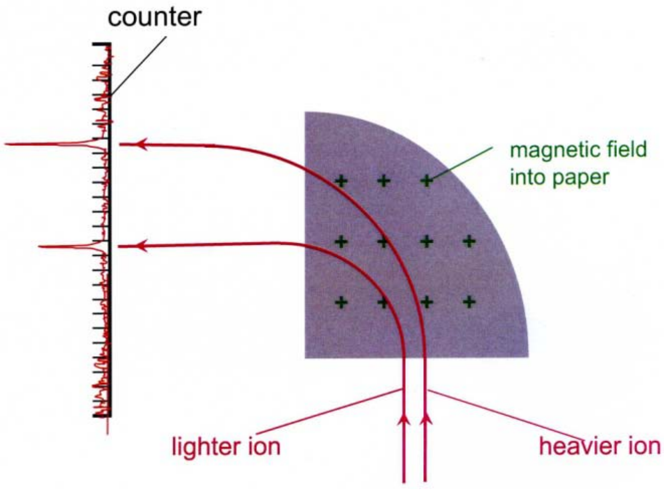


Figure 5.10 Schematic illustrating the principle of operation of a mass spectrometer

on a detector at the exit of the spectrometer.

Example: In a mass spectrometer the velocity (v) of a stream of particles, travelling with a radius of 40 cm in a magnetic field of 0.36 T, was measured at $v = 6 \times 10^5$ ms^{-1}. Assuming that the particles are singly charged ($q = 1.6 \times 10^{-19}$ C), work out what the substance is.

[Avagadro's number = 6×10^{23} particles per mole]

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow m = \frac{qvBr}{v^2} = \frac{qBr}{v}$$

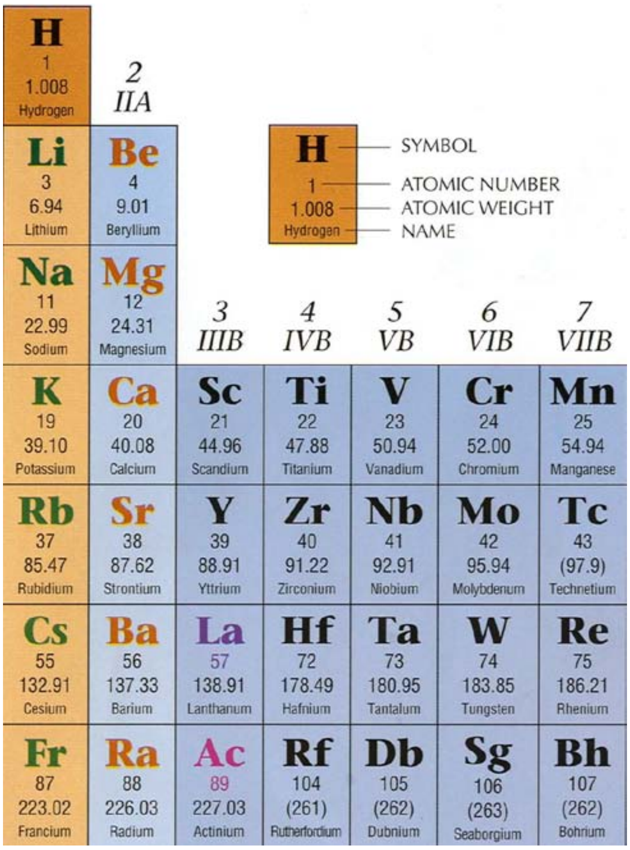
$$\Rightarrow m = \frac{1.6\times10^{-19}\times0.36\times0.4}{6\times10^5} = 3.84\times10^{-26}\ \text{kg}$$

Now, the atomic weight of a substance is the weight, in grams, of one mole (i.e. 6×10^{23} particles) of that substance.

One particle of our 'mystery' substance has a mass of 3.84×10^{-23} grams, so one mole of the substance has a weight of

$3.84 \times 10^{-23} \times 6 \times 10^{23} = 23.0$ gmol^{-1}

We can now just look up on a periodic table of the elements what substance has an atomic weight of 23 – and the answer is Sodium.



5.6 Torque on a coil in a magnetic field.

In the previous sections we have seen how a conductor carrying a current in a magnetic field is subjected to a force. In this section we will look at how these forces, in a loop of wire or a coil, can combine to produce a turning force or *torque*. The rotation of a coil in a magnetic field due to the current passing through it is obviously something that is of huge technological significance, being the basis of many devices such as the moving coil galvanometer and, most importantly, the electric motor.

Let us start by considering the direction of the forces experienced by a simple loop of wire in a magnetic field, as shown in Figure 5.11.

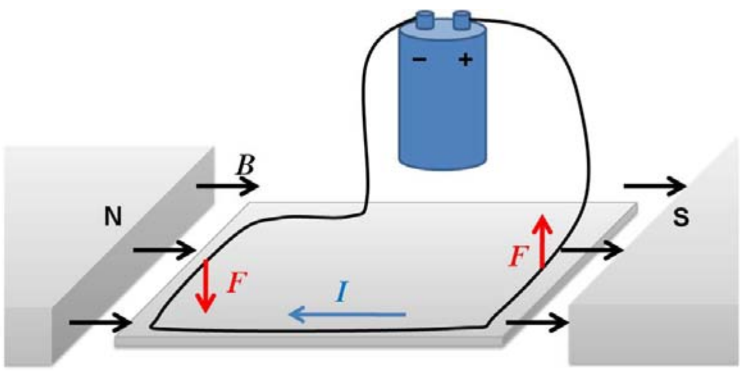


Figure 5.11 The forces on a current-carrying loop in a magnetic field

Using the right-hand rule it is relatively easy to establish the direction of the forces on the loop shown in the diagram. It can be seen that on the two sides of the loop that are perpendicular to the field, the forces act in opposite directions and, if it were free to, there would be a tendency for the loop to rotate.

So, now let's see how these forces work when applied to a coil that *is* free to rotate.

Figure 5.12(a) shows a rectangular coil that is free to rotate about its central vertical axis. The axis is at right angles to the direction of a uniform magnetic field of flux density B.

Figure 5.12(b) shows the top view of the coil when its plane is parallel to the field. The cross and dot show the current direction through the *vertical* sides of the coil. The direction of the force F acting on each vertical side is drawn in accordance with the right hand rule. The current through the *horizontal* sides of the coil, at the top and bottom, is parallel to the field direction and therefore the magnetic force acting on them is zero. The forces acting on the vertical sides constitute a couple and the coil rotates until its plane is perpendicular to the field direction, as shown in Figure 5.12(c). The forces acting on the vertical sides are still the same as before but their lines of action both pass through the vertical axis, so there is no further tendency to rotate. The right-hand rule shows that there are now forces acting on the horizontal sides, but they are directed vertically up and down and have no effect on the rotation of the coil.

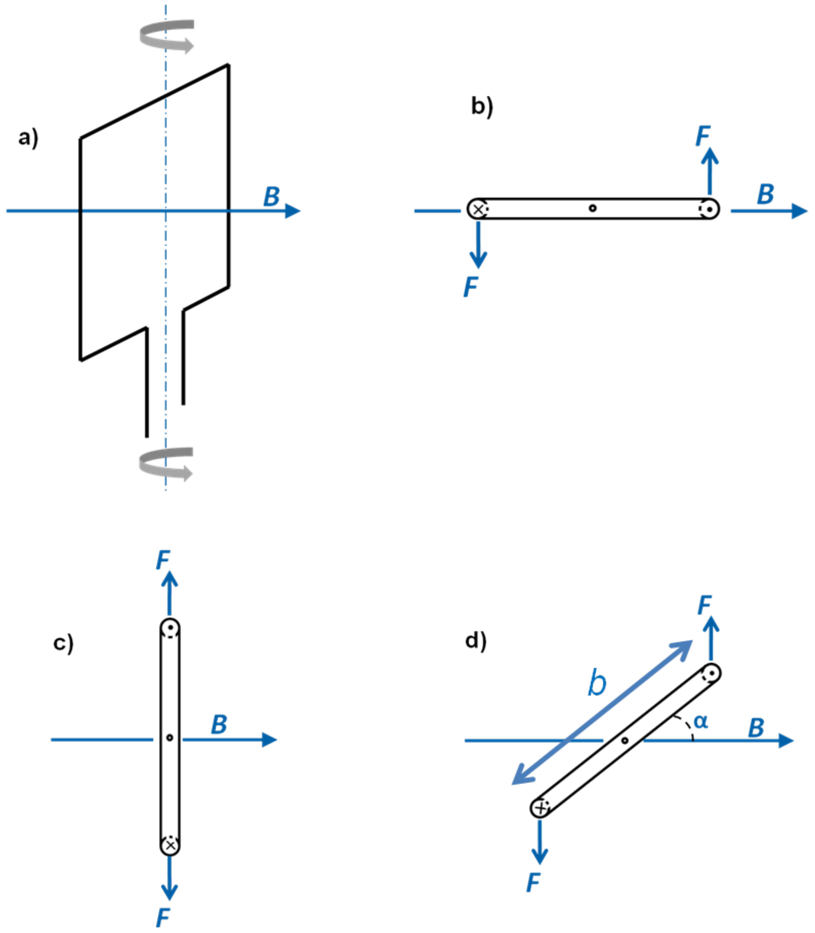


Figure 5.12 Torque on a rotating coil in magnetic field

Figure 5.12(d) shows the coil when it is inclined at an angle α to the field direction. Considering the moments[43] of the forces involved, we can say that the torque T due to the couple about the axis of rotation is given by

$$T = F \times b\cos\alpha \qquad \text{[newtons, N]}$$

where b is the width of the coil and $b\cos\alpha$ is the perpendicular distance between the lines of action of the forces F.

[43] Moment of force (often just *moment*) is the tendency of a force to twist or rotate an object

The vertical sides of the coil remain perpendicular to the direction of the field, whatever the value of α; therefore, F always has the same value i.e BIL (where L is the length of the vertical sides). If the coil has N turns, then, in effect, L is multiplied by N and F is given by

$$F = BINL$$

Substituting this into the equation for torque (above) gives

$$T = BINLb\cos\alpha$$

But Lb is just the area of the coil face, therefore

$$T = BINA\cos\alpha \qquad (5.5)$$

Note that when $\alpha = 0°$, then $\cos\alpha = 1$ and $T = BINA$, and if $\alpha = 90°$, then $\cos\alpha = 0$ and $T = 0$ – as in Figure 5.12(b) and (c) respectively.

5.7 Electric motors

We have already said several times that the force acting on a current-carrying coil in a magnetic field is the basis of the electric motor, in which electrical energy is converted into mechanical energy.

Electric motor designs are numerous – and it is beyond the scope and aims of this course to look at such designs in detail. However, there are a couple of general principles that are worth noting briefly. At this stage we will restrict our observations to DC motors.

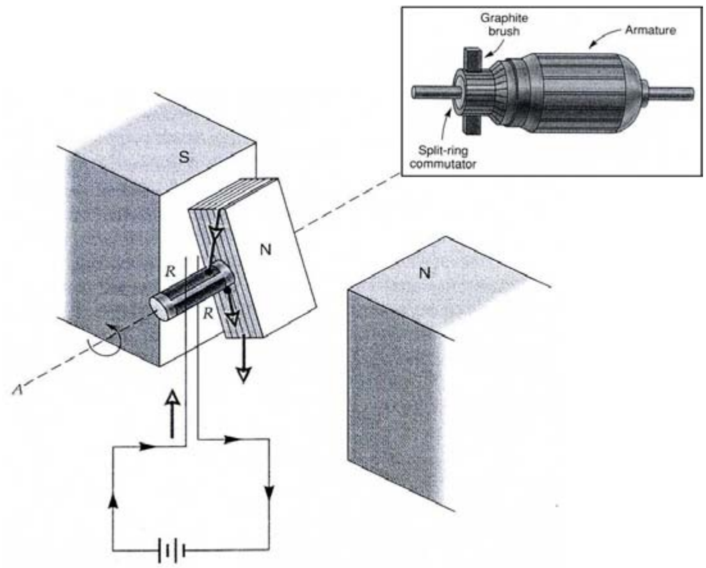


Figure 5.13 Schematic of simple single-coil DC motor and (inset) a multi-coil motor.

As can be seen by looking at Figure 5.13, the coil of an electric motor will have its own magnetic field due to the current flowing through it. This means that it will have an effective north and south pole. As a consequence, when the coil rotates there will come a point when the north pole of the motor coil is facing the south pole of the motor magnet – and the attraction between the opposite poles will tend to stop the rotation.

In order to overcome this problem, *split-ring* contacts are used to connect to the coil. These are arranged such that, every half-turn of the coil, the current direction in the coil, and so the induced magnetic field, is reversed. This prevents opposite poles of the magnets coming together.

Our analysis of the torque on a rotating coil in a magnetic field showed that it is not constant; it varies as the coil rotates, from a minimum of zero to a maximum of $BINA$ newtons. Clearly, this means that the torque produced by an electric motor is not constant. At certain points the torque is actually equal to zero and the motor relies on momentum to get it through this point and to continue rotating. This unevenness in the torque can be a problem, especially when it comes to starting the motor under load.

One way of resolving this problem of uneven torque is to have more than one coil wound on the rotating shaft (*rotor*), of the motor (illustrated in the inset to Figure 5.13). If multiple coils are wound onto the rotor at different angles, it means that when one coil is experiencing reduced (or zero) torque, one of the other coils will be experiencing increased torque. Such an arrangement does require a more complicated arrangement of split-rings (the *split-ring commutator*) so that the appropriate coils are contacted at the right time.

5.8 Forces between parallel conductors

Two straight current-carrying conductors placed parallel to one another each experience a force due to the magnetic field of the other. The forces are attractive if the currents are flowing in the same direction and repulsive if they are in opposite directions.

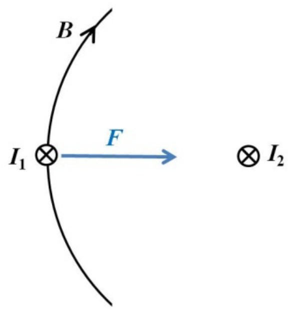


Figure 5.14 The force on parallel current-carrying conductors

Figure 5.14 shows two parallel conductors running perpendicularly through the page, each carrying a current downwards into the paper. The current I_2 in the right-hand conductor produces a field B at the left-hand conductor, as shown. The current I_1 flowing through this field therefore experiences a force (according to the right-hand rule) that acts to the right (towards the other conductor). Similarly, the right-hand conductor experiences a force to the left because of the magnetic field due to the current I_1 in the left-hand conductor.

Similar arguments show that the forces between the conductors are *repulsive* if the currents flow in *opposite directions* to one another.

The SI unit of current, the ampere, is defined as the steady current in each of two straight, parallel conductors of infinite length and negligible cross-sectional area, 1 metre apart in vacuum, that produces a force between them of 2×10^{-7} N per metre length.

Problem 8:

A 12m length of metal wire, with a density of 8900 kgm^{-3} and a resistivity of 1.7×10^{-8} Ωm, is aligned horizontally in a west-east direction. A potential difference of 890 V across the ends of the wire provides just enough support for its weight. Estimate the magnitude of the earth's magnetic field acting horizontally at that point. [Assume g = 9.8 ms^{-2}).