3. $y'' + \frac{1}{\sin x} y' + \frac{1-x}{x^2} y = 0$

$$\lim_{x \to n\pi} \frac{x - n\pi}{\sin x} = \frac{1}{\cos(n\pi)} = (-1)^n$$

$x = n\pi$ where n is an integer are regular singular points.

Using the method of Frobenius

$$y = \sum_{n=0}^{\infty} c_n x^{n+r} \qquad y' = \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} \qquad y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-2}$$

$$\sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-2} + \frac{1}{\sin x} \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} + \frac{1-x}{x^2} \sum_{n=0}^{\infty} c_n x^{n+r}$$

$$x^2 \sin x \, y'' + x^2 y' + (1-x) \sin x \, y = 0$$

$$\left(\text{Remember } \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \right)$$

Plug into DE:

$$x^r \left[\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^n + \sum_{n=0}^{\infty} (n+r) c_n x^{n+1} + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \sum_{n=0}^{\infty} c_n x^n \right.$$

$$\left. - \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \sum_{n=0}^{\infty} c_n x^{n+1} \right]$$

$\downarrow$?