

Problem 6 – due at start of class on Friday 10/08.

A planet of unknown mass is in a circular orbit of radius R around a massive star. The planet moves at constant speed and completes one orbit in a known time T . In terms of the known quantities and the universal constant G , determine the mass of the star.

Your solution should contain all the following elements:

1. A sketch showing a star, a planet and a circular orbit with the radius of the orbit indicated and labeled R .
2. A list defining all symbols you will use, including those that are not given in the statement of the problem but which will be required in the solution. For example, you will need the mass of the star and the mass of planet. You do not need to define G – this symbol is always used for the universal gravitational constant. You should leave some space here. As you develop your solution you will probably find that you need to define additional symbols.
3. An algebraic expression for $\left| \frac{d\vec{p}}{dt} \right|$ in terms of defined variables. *Hint:* What is $\left| \frac{d\vec{p}}{dt} \right|$ for an object in a circular orbit at constant speed?
4. A statement that the net force on the planet is the gravitational interaction with the star.
5. An algebraic expression for the gravitational force on the planet in terms of defined variables.
6. Use the derivative form of the momentum principle to obtain $\left| \frac{d\vec{p}}{dt} \right| = |\vec{F}_{net}|$ and note that because this is a circular orbit at constant speed there is no parallel component to the force.
7. Algebraic manipulation of the above equation to get an expression for the mass of the star in terms of the known quantities. Make sure your work is logically structured with the correct placement of equal signs and comments where you make substitutions. (For example, at some point you will probably have to substitute $v = \frac{2\pi R}{T}$ where v is the speed of the planet)

Checks:

- Does your final solution only involve, R , T and other known quantities like G , π , etc.
- Although you have an algebraic expression you can still check your units. Write down the SI units for each term in your expression, cancel all the units that you can and check that the end result is the SI unit for mass. *Hint:* remember to write N as $\text{kg}\cdot\text{m}\cdot\text{s}^{-2}$
- Check that your vector signs are correct. Make sure that you don't have any vectors equal to scalars.
- Check the logical flow of your solution.