

$$x = \cos^2(t)$$

$$y = \cos(t)$$

$$\begin{aligned} \int_0^\pi \sqrt{(2\cos(t)(-\sin(t)))^2 + (-\sin(t))^2} dt &= \int_0^\pi \sqrt{(-2\cos(t)\sin(t))^2 + \sin^2(t)} dt \\ &= \int_0^\pi \sqrt{4\cos^2(t)\sin^2(t) + \sin^2(t)} dt = \int_0^\pi \sqrt{\sin^2(t)(4\cos^2(t) + 1)} dt \\ &= \int_0^\pi \sin(t) \sqrt{4\cos^2(t) + 1} dt \end{aligned}$$

$$\text{Let } u = \cos(t)$$

$$du = -\sin(t)dt$$

$$= -\int_0^\pi \sqrt{4u^2 + 1} du$$

$$u = \frac{\tan s}{2}, du = \frac{\sec^2 s}{2}$$

$$= -\int_0^\pi \sqrt{4\left(\frac{\tan s}{2}\right)^2 + 1} \left(\frac{\sec^2 s}{2}\right) = -\int_0^\pi \sqrt{\sec^2 s} \left(\frac{\sec^2 s}{2}\right) = -\int_0^\pi \left(\frac{\sec^3 s}{2}\right)$$

$$= -\frac{1}{2} \int_0^\pi \sec^3 s = -\frac{1}{2} \int_0^\pi \sec^2 s * \sec(s)$$

$$u = \sec(s), dv = \sec^2 s$$

$$du = \sec(s)\tan(s), v = \tan(s)$$

$$-\frac{1}{2} \int_0^\pi \sec^3 s * ds = \sec(s)\tan(s) \Big|_0^\pi - \int_0^\pi \sec(s)\tan^2(s)$$

$$-\frac{1}{2} \int_0^\pi \sec^3 s * ds = \sec(s)\tan(s) \Big|_0^\pi - \int_0^\pi \sec(s)(\sec^2(s) - 1)$$

$$= -\int_0^\pi \sec^3(s) - \sec(s)ds = -\int_0^\pi \sec^3(s) + \ln(\sec(s) + \tan(s)) \Big|_0^\pi$$

$$\frac{1}{2} \int_0^\pi \sec^3 s = \sec(s)\tan(s) \Big|_0^\pi + \ln(\sec(s) + \tan(s)) \Big|_0^\pi$$

$$\int_0^\pi \sec^3 s = 2 \left(\sec(s)\tan(s) \Big|_0^\pi + \ln(\sec(s) + \tan(s)) \Big|_0^\pi \right)$$

$$= 2 \left(\sec(s)\tan(s) \Big|_0^\pi + \ln(\sec(s) + \tan(s)) \Big|_0^\pi \right)$$

$$\sec(s) = 2\cos(t)^2 + 1^2$$

$$\tan(s) = 2\cos(t)$$

$$= 2 \left(\left((4\cos^2(t) + 1)(2\cos(t)) \right) \Big|_0^\pi + \ln \left((4\cos^2(t) + 1) + (2\cos(t)) \right) \Big|_0^\pi \right)$$

$$= 2 * 6 \left(\left((4\cos^2(t) + 1)(2\cos(t)) \right) \Big|_0^\pi + \ln \left((4\cos^2(t) + 1) + (2\cos(t)) \right) \Big|_0^\pi \right)$$

$$= 12 * (-20 + \ln(3) - \ln(7))$$

$$\approx -250.167$$