I have asked to find $Y_{1-1}(\theta,\varphi)$ (l=1,m=-1). I know that:

$$P_l(\xi) = \frac{1}{2^l l!}\frac{d^l}{d\xi^l}(\xi^2-1)^l$$

$$P_l^m(\xi) = (1-\xi^2)^{m/2}\frac{d^m}{d\xi^m}P_l(\xi) \quad (0 \leq m \leq l)$$

$$Y_{lm}(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}(-1)^m P_l^{|m|}(\cos\theta)\exp(\imath m\phi)$$

So as I understand if I take $\xi = \cos\theta$ in the first equation from above I'll get:

$$P_1(\cos\theta) = \frac{1}{2}\frac{d}{d(\cos\theta)}(\cos^2\theta - 1) = \frac{1}{2}\cdot 2\cos\theta = \cos\theta$$

Assaying it in the second equation:

$$P_1^{|-1|}(\cos\theta) = (1-\cos^2\theta)^{1/2}\frac{d}{d(\cos\theta)}P_1(\cos\theta) = \sin\theta\frac{d}{d(\cos\theta)}\cos\theta = \sin\theta$$

Finally adding the finale result to the third equation I get:

$$Y_{1-1}(\theta,\varphi) = \sqrt{\frac{3}{4\pi}\frac{(2)!}{(0)!}}\cdot(-1)^{-1}\cdot P_1^{|-1|}(\cos\theta)\cdot\exp(im\varphi) = -\sqrt{\frac{3}{2\pi}}\cdot\sin\theta\cdot\exp(-i\varphi)$$

But the know Result is:

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\, e^{-i\varphi}$$

I would appreciate if someone could explain we what went wrong with my solution.

Thanks for your time,

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