

5. Let  $\mathbf{Z} * \mathbf{Z}_2 = \langle a, b \mid b^2 \rangle$  be represented by  $X = S^1 \vee P^2$

For the subgroup  $H$  below construct the covering space  $\tilde{X}$  by sketching a good picture for  $\tilde{X}$  and explaining (in your picture) how it covers  $X$ .

In each case give a group presentation for the group  $G$  of covering transformations of the covering  $p: \tilde{X} \rightarrow X$  and describe (using your picture) the action of  $G$  on  $\tilde{X}$ .

(a)  $H$  is the smallest normal subgroup containing  $b$ .

(a)  $H$  is the smallest normal subgroup containing  $a$ .

(a)  $H$  is the smallest normal subgroup containing  $a^2$  and  $b$ .

(a)  $H$  is the trivial subgroup.