A Theory of Gravity Based on Special Relativity

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Abstract: We obtain the effects of time dilation and length contraction starting with the law of universal gravitation and the influence of gravitational field on the light. Covariant theory of gravity is derived from law of universal gravitation and special relativity. We calculate the standard tests of general relativity and find that the results basically agree with that of general relativity. However, our theory differs from general relativity in the predictions for GP-B test, which expects the geodetic effect to be zero and frame-dragging effect 1/4 of the result in general relativity. The anomalous precession of DI Herculis and origin of quasars are also explained in the paper.

Keywords: Law of universal gravitation, Special relativity, General relativity

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1 Introduction

Einstein's general relativity is a generally covariant theory of gravity, which encompasses the law of universal gravitation in its lower order approximation. The following theory starts with the law of universal gravitation and makes it a covariant theory by combining it with special relativity. For the standard tests of general relativity, our theory also gets the same results. Fortunately, GP-B test offers an opportunity to distinguish between the two theories, because our predictions are different from that of general relativity. In addition, our theory is much simpler than general relativity in forms.

2 The mass variation of body moving in static gravitational field

We know that Coulomb's law applies to the interactions between rest charges, but can be extended to the interaction of rest charge on a moving charge. Similarly, we suppose Newton's law of universal gravitation also holds for the interaction of rest gravitational source on a moving body.

We suppose, for simplicity, that a point mass M is fixed at point o, as shown in Figure 1, and a point particle with the rest mass of m_0 is falling freely in the static gravitational field produced by M. As it moves, the work done on it by the gravitational field will be converted into its kinetic energy, and its inertial mass increases. Then according to the principle of equivalence, its gravitational mass increases and it experiences a larger gravitational force.

We suppose that the particle m moves a differential distance of dr, and the change of mass is dm. Based on the relationship of work-energy, we have

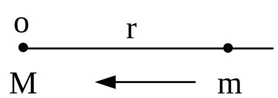

Figure 1. Particle falls freely in static gravitational field

$$dmc^2 = -\frac{GMm}{r^2}dr \tag{1}$$

$$\frac{dm}{m} = -\frac{GM}{r^2c^2}dr \tag{2}$$

Integrating on both sides of the equation, we find

$$m = ke^{\frac{GM}{rc^2}} \tag{3}$$

where k is constant. As particle falls at rest from infinity, we have $k = m_0$. If particle is released at rest from the distance of r_0, we have $k = m_0 e^{-GM/r_0c^2}$.

We know that the mass of photon is $\hbar\omega/c^2$. Suppose the frequency of light at infinity is ω_0. As it travels in the gravitational field, according to Equation (3), its frequency will change to $\omega = \omega_0 e^{\frac{GM}{rc^2}} \approx \omega_0(1+\frac{GM}{rc^2})$, which agrees in first order approximation with the result $\omega = \omega_0/\sqrt{1-2GM/rc^2} \approx \omega_0(1+\frac{GM}{rc^2})$ in general relativity. Likewise, if the frequency of light at the surface of a star is ω_0, it becomes $\omega = \omega_0 e^{-GM/Rc^2} \approx \omega_0(1-GM/Rc^2)$ as it propagates to infinity, which is the formula of gravitational red shift.

We know that the expression of gravitational potential is $-GM/r$ in Newton dynamics. Now we calculate the correct formula of gravitational potential energy. Based on the definition, the potential energy of body equals to the negative value of the work done by the gravitational field as it moves from infinity. Thus we have

$$E_p = \int_\infty^r \frac{GMm}{r^2}dr = \int_\infty^r \frac{GMm_0}{r^2}e^{\frac{GM}{rc^2}}dr = m_0c^2(1-e^{\frac{GM}{rc^2}}) \tag{4}$$

3 The motion of body in static gravitational field

3.1 The effects of time dilation and length contraction in gravitational field

As light travels in gravitational field, its speed at each point is different. Thus we cannot define synchronous clocks as in special relativity. As pointed out in Ref [1], we cannot observe time dilation effect by merely measuring the interval of the ticks of a clock and comparing it with the time standard defined by the maker, because the gravitational influence on the time standard is equal to that on the clock. That is to say, if a clock reads one second for a physical process without gravitational influence, it will still read one second in the presence of gravitational field, because clock and process are affected with gravitational field in the same manner.

In order to synchronize the clocks at different positions in gravitational field, we compare the rest clock in gravitational field with the clock at infinity. We may think that the gravitational force at infinity is zero and in this inertial system the speed of light is c. Suppose a sequence of oscillating wave propagates from infinity to r, whose frequency is ω_0 and the interval between two adjacent wave crests is $\Delta t = 2\pi / \omega_0$ at infinity. Then when measured by the rest clock in the gravitational field the interval is still Δt, because the delays needed for the two wave crests to travel from infinity to r are the same. This is just as what we have pointed out in the above that gravitational field has the same influence on the clock and the process. But it can be seen from our above calculation that the local frequency of the light now becomes $\omega_0 e^{\frac{GM}{rc^2}}$, which is relative to the frequency at infinity. That is to say, when observed from the distant inertial frame, the local frequency becomes $\omega_0 e^{\frac{GM}{rc^2}}$ and the local interval between two adjacent wave crests is $e^{-\frac{GM}{rc^2}}\Delta t$. Now that the interval measured by the local clock is Δt, the clock rested in gravitational field slows down. In order to be consistent with the clock at infinity we must multiply the interval that the local clock measures by a factor of $e^{-\frac{GM}{rc^2}}$.

Now we turn to the length contraction effect in gravitational field. Suppose we have a ruler and a pole. When there is no gravitational field, the length of the pole that the ruler measures is one meter. Then in gravitational field, if we place the ruler and the pole in the radial direction of the gravitational field, the length measured by the ruler is still one meter, i.e., the ruler and the pole experience the same contraction. In order to see the length contraction effect in gravitational field, we suppose the wavelength of light at infinity is λ_0 and the distance it travels within unit time is $n\lambda_0$. As it propagates to r, there are still n waves within unit time. Now that the local frequency increases, the local wavelength becomes $e^{-\frac{GM}{rc^2}}\lambda_0$ when observed from infinity, and the distance that light travels within unit time is $e^{-\frac{GM}{rc^2}}n\lambda_0$. We define length to be the distance that light travels within certain time. Thus compared with the length at infinity, the local length contracts. In order to be consistent with the length at infinity, we should multiply the local length in the radial direction of gravitational field by a factor $e^{\frac{GM}{rc^2}}$.

Note that the length in the tangential direction of the gravitational field does not contract, because light will be unaffected with gravitational field as it travels in the tangential direction.

We summarize as follows. In order to establish synchronous time standard and unified length standard in gravitational field, we must multiply the local time interval by a factor of $e^{-\frac{GM}{rc^2}}$ and the local radial length a factor of $e^{\frac{GM}{rc^2}}$. In doing so, the clocks and the rulers in gravitational field will be consistent respectively with the clock and ruler at infinity.

3.2 The motion equations in static gravitational field

In general relativity, Schwarzschild metric is used to solve the motion problem in static gravitational field. We see that considering the effects of time dilation and length contraction, the expression of spherically symmetrical line element in static gravitational field can be written

$$ds^2 = c^2 e^{-\frac{2GM}{rc^2}} dt^2 - e^{\frac{2GM}{rc^2}} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2 \tag{5}$$

which in first order approximation becomes Schwarzschild metric

$$ds^2 = c^2 (1 - \frac{2GM}{rc^2}) dt^2 - (1 - \frac{2GM}{rc^2})^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2 \tag{6}$$

The motion equations in static gravitational field can be obtained from variation principle by solving the following equation

$$\delta \int ds = 0 \tag{7}$$

For the motion of light, there is another equation

$$ds = 0 \tag{8}$$

The above method is applied in general relativity. Now we want to obtain the motion equations by directly applying the conservation laws of angular momentum and energy, whose physical meanings are very explicit to us.

We first write down the equations of angular momentum and energy conservation without gravitational field. Then we make corresponding substitutions based on the effects of time dilation and length contraction in gravitational field. The equations of angular momentum and energy conservation without gravitational field can be written (suppose the motion takes place in the plane of $\theta = \frac{\pi}{2}$)

$$\frac{m_0}{\sqrt{1-\beta^2}} r^2 \frac{d\varphi}{dt} = L_0 \tag{9}$$

$$\frac{m_0 c^2}{\sqrt{1-\beta^2}} - m_0 c^2 = E_0 \tag{10}$$

where $\beta^2 = \frac{1}{c^2}[(\frac{dr}{dt})^2 + r^2 (\frac{d\varphi}{dt})^2]$. In the presence of gravitational field, we should make such substitutions: $r \to re^{GM/rc^2}$, $dr \to dre^{GM/rc^2}$, $dt \to dte^{-GM/rc^2}$, $rd\varphi \to rd\varphi$, and now energy includes kinetic energy and potential energy. Thus we get the equations of angular momentum and energy conservation as

$$\frac{m_0}{\sqrt{1-\beta^2}} e^{\frac{2GM}{rc^2}} r^2 \frac{d\varphi}{dt} = L_0 \tag{11}$$

$$\frac{m_0 c^2}{\sqrt{1-\beta^2}} - m_0 c^2 + m_0 c^2 (1 - e^{\frac{GM}{rc^2}}) = E_0 \tag{12}$$

where $\beta^2 = \frac{1}{c^2}[e^{\frac{4GM}{rc^2}} (\frac{dr}{dt})^2 + e^{\frac{2GM}{rc^2}} r^2 (\frac{d\varphi}{dt})^2]$. For the motion of light, we should substitute following equation for Equation (12).

$$e^{\frac{4GM}{rc^2}} (\frac{dr}{dt})^2 + e^{\frac{2GM}{rc^2}} r^2 (\frac{d\varphi}{dt})^2 = c^2 \tag{13}$$

3.3 The perihelion advance of the Mercury

Now we solve the perihelion advance problem for the Mercury using above equations. As $\beta << 1$, we simplify Equations (11) and (12) as

$$m_0 e^{\frac{2GM}{rc^2}} r^2 \frac{d\varphi}{dt} = L_0 \tag{14}$$

$$\frac{1}{2} m_0 [e^{\frac{4GM}{rc^2}} (\frac{dr}{dt})^2 + e^{\frac{2GM}{rc^2}} r^2 (\frac{d\varphi}{dt})^2] + m_0 c^2 (1 - e^{\frac{GM}{rc^2}}) = E_0 \tag{15}$$

From Equation (14), we obtain $\frac{d\varphi}{dt} = \frac{L_0}{m_0 r^2} e^{-\frac{2GM}{rc^2}}$. Substituting it into Equation (15), we have

$$\frac{L_0^2}{m_0^2 r^4} (\frac{dr}{d\varphi})^2 + \frac{L_0^2}{m_0^2 r^2} e^{-\frac{2GM}{rc^2}} - 2c^2 (1 - e^{\frac{GM}{rc^2}}) = \frac{2E_0}{m_0} \tag{16}$$

Let $u = 1/r$. Expanding $e^{\frac{GM}{rc^2}}$ in series, we get

$$(\frac{du}{d\varphi})^2 + u^2 - \frac{2GM}{c^2} u^3 - \frac{2GMm_0^2}{L_0^2} u - \frac{G^2 M^2 m_0^2}{c^2 L_0^2} u^2 - \frac{1}{3} \frac{G^3 M^3 m_0^2}{c^4 L_0^2} u^3 = \frac{2E_0 m_0}{L_0^2} \tag{17}$$

As the latter two terms that comprise u^2 and u^3 respectively are much less than the former two and can be ignored, we obtain

$$(\frac{du}{d\varphi})^2 + u^2 - \frac{2GM}{c^2} u^3 - \frac{2GMm_0^2}{L_0^2} u = \frac{2E_0 m_0}{L_0^2} \tag{18}$$

Differentiating the equation with respect to φ, we have

$$\frac{d^2 u}{d\varphi^2} + u - \frac{3GM}{c^2} u^2 = \frac{GMm_0^2}{L_0^2} \tag{19}$$

Now we get the same equation as in general relativity [2],[3]. We apply the method used in Ref [2], let $u = GMm_0^2 / L_0^2 + u_1$ and substitute it into above equation. In first order approximation, we get

$$\frac{d^2u_1}{d\varphi^2}+u_1(1-\frac{6G^2M^2m_0^2}{L_0^2c^2})=0 \tag{20}$$

whose solution is $u_1=k\cos\sqrt{1-\frac{6G^2M^2m_0^2}{L_0^2c^2}}\varphi$. Thus we get

$$u=\frac{GMm_0^2}{L_0^2c^2}+k\cos\sqrt{1-\frac{6G^2M^2m_0^2}{L_0^2c^2}}\varphi \tag{21}$$

For the above equation, we see that when φ revolves 2π, Mercury must advance another value Δ can u return to its initial value at $\varphi=0$. We expand the square root and get

$$(1-\frac{3G^2M^2m_0^2}{L_0^2c^2}+\cdots\cdots)(2\pi+\Delta)=2\pi \tag{22}$$

$$\Delta\approx\frac{6\pi G^2M^2m_0^2}{L_0^2c^2} \tag{23}$$

We now solve for the orbital angular momentum L_0. As the angular momentum is conservative, we calculate L_0 at perihelion for simplicity. At perihelion we have

$$\frac{mv^2}{R}=\frac{GMm}{r^2} \tag{24}$$

where R is the radius of curvature at perihelion. The orbit of the Mercury is ellipse, which can be written

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{25}$$

where a,b are semimajor axis and semiminor axis, respectively. The expression of the radius of curvature is

$$R=\frac{(1+y'^2)^{3/2}}{|y''|} \tag{26}$$

So at perihelion we find $R=b^2/a=a(1-e^2)$, where e is eccentricity. The angular momentum of Mercury at perihelion is

$$L_0=m_0r^2\frac{d\varphi}{dt}=m_0vr=m_0\sqrt{GMR}=m_0\sqrt{GMa(1-e^2)} \tag{27}$$

Substituting it into Equation (23), we find

$$\Delta\approx\frac{6\pi GM}{c^2a(1-e^2)} \tag{28}$$

which agrees with the result of general relativity [1],[3]. From Kepler's second law, we obtain [4]

$$2\pi ab = hT \tag{29}$$

where h is the angular momentum for unit mass, i.e., $h = L_0 / m_0$. Substituting it into Equation (27), we have

$$GM = \frac{h^2}{a(1-e^2)} = \frac{4\pi^2 a^2 b^2}{T^2 a(1-e^2)} = \frac{4\pi^2 a^3}{T^2} \tag{30}$$

Substituting it into Equation (28), we find

$$\Delta \approx \frac{24\pi^3 a^2}{c^2 T^2 (1-e^2)} \tag{31}$$

Maybe someone will be suspicious that whether it is appropriate to use the potential energy expression of Equation (4) in above equations since Mercury cannot run to infinity. In fact, the zero point of potential energy can be taken anywhere. For the motion of Mercury, we can choose any point in orbit as the zero point of potential energy. For example, we choose aphelion r_0 as the zero point of potential energy. Then we use the relation between work and energy to derive the equation of energy conservation. As Mercury moves from aphelion r_0 to r, the increase of the kinetic energy is $\frac{m_0 c^2}{\sqrt{1-\beta^2}} - \frac{m_0 c^2}{\sqrt{1-\beta_0^2}}$, and the work done by the gravitational field is

$$W = \int_{r_0}^{r} -\frac{GMm}{r^2} dr = \int_{r_0}^{r} -\frac{GMk}{r^2} e^{\frac{GM}{rc^2}} dr = kc^2 (e^{\frac{GM}{rc^2}} - e^{\frac{GM}{r_0 c^2}}) \tag{32}$$

We now solve for the constant k. As Mercury is at $r = r_0$, its mass is $m_0 / \sqrt{1-\beta_0^2}$. From Equation (3), we have

$$\frac{m_0}{\sqrt{1-\beta_0^2}} = k e^{\frac{GM}{r_0 c^2}} \tag{33}$$

$$k = \frac{m_0}{\sqrt{1-\beta_0^2}} e^{-\frac{GM}{r_0 c^2}} \tag{34}$$

As Mercury moves from aphelion r_0 to r, the increase of the kinetic energy equals to the work done by the gravitational field. Thus we have

$$\frac{m_0 c^2}{\sqrt{1-\beta^2}} - \frac{m_0 c^2}{\sqrt{1-\beta_0^2}} = \frac{m_0 c^2}{\sqrt{1-\beta_0^2}} (e^{\frac{GM}{rc^2} - \frac{GM}{r_0 c^2}} - 1) \tag{35}$$

For the motion of Mercury, Equation (12) should be replaced by the above equation. But it can be seen that insignificant differences exist between them in weak gravitational field and they lead to the same precession magnitude.

It should be noted that Equation (28) or (31) only holds for the motion of planets about the Sun. In this case, the masses of the planets are much less than that of the Sun and we may think that the Sun is at rest and it is an inertial frame. In the case of binary system, the masses of two component stars are matched and they both move round the center of mass. As the center of mass of binary is an inertial frame, we should observe the motion of binary from the center of mass.

For binary system, the center of mass must be in the line that connects the binary stars. Suppose the distance from the center of mass C to m is r_1, and to M r_2, we have $mr_1 = Mr_2$. The motion equation for m with respect to C is

$$m\ddot{r}_1 = -\frac{GMm}{(r_1 + r_2)^2}\frac{\mathbf{r}_1}{r_1} \tag{36}$$

Substituting $r_2 = \frac{m}{M} r_1$ into above equation, we have

$$m\ddot{r}_1 = -\frac{GM^3 m}{(m+M)^2}\frac{1}{r_1^2}\frac{\mathbf{r}_1}{r_1} \tag{37}$$

which is equivalent that there exists a gravitational source with the mass of $\frac{M^3}{(m+M)^2}$ while the other component star does not exist. Compared with the instance of solar system, M is equivalent to be $\frac{M^2}{(m+M)^2}$ times smaller. Thus Equation (28) or (31) must be multiplied by a factor of $\frac{M^2}{(m+M)^2}$ when we calculate the precession of the binary system.

For DI Herculis, $m = 4.52 M_S$, $M = 5.15 M_S$, where M_S is the mass of the Sun. For the component star of m, $e = 0.489$, $T = 10.55d$, $a = 43.2 R_s$, where R_s is the radius of the Sun. The cumulative precession magnitude in 100 years according to Equation (31) is 2.34°. Thus the actual precession value is $2.34^\circ \times \frac{5.15^2}{(4.52+5.15)^2} = 0.66^\circ$, which is in good agreement with the observed value of $0.65 \pm 0.18^\circ$/100 yr [5].

3.4 The light deflection

For the motion of light in gravitational field, if we regard light as a particle with unit mass, we can use Equations (11) and (13) to calculate the light deflection.

$$e^{\frac{2GM}{rc^2}} r^2 \frac{d\varphi}{dt} = L_0 \tag{38}$$

$$e^{\frac{4GM}{rc^2}} \left(\frac{dr}{dt}\right)^2 + e^{\frac{2GM}{rc^2}} r^2 \left(\frac{d\varphi}{dt}\right)^2 = c^2 \tag{39}$$

Following the similar steps in the above we get

$$\frac{d^2u}{d\varphi^2}+u-\frac{3GM}{c^2}u=0 \tag{40}$$

which agrees with the equation in general relativity [2],[3]. The bending angular of light at the limb of the Sun is

$$\alpha=\frac{4GM}{Rc^2} \tag{41}$$

3.5 The delay of radar echo

For the delay of radar echo, we solve for $d\varphi/dt$ from Equation (38) and substitute it into Equation (39). Then we get the expression of dr/dt as

$$\frac{dr}{dt}=\sqrt{c^2e^{-\frac{4GM}{rc^2}}-\frac{L_0^2}{r^2}e^{-\frac{6GM}{rc^2}}} \tag{42}$$

At perihelion $r=r_{\min}$, we have $\frac{dr}{dt}=0$. From Equations (38) and (39) we obtain $L_0=cr_{\min}e^{\frac{GM}{r_{\min}c^2}}$. Substituting it into above equation, we get

$$\frac{dr}{dt}=c(1-\frac{2GM}{rc^2})\sqrt{1-\frac{r_{\min}^2}{r^2}(1+\frac{2GM}{r_{\min}c^2})(1-\frac{2GM}{rc^2})} \tag{43}$$

The result in general relativity is [6]

$$\frac{dr}{dt}=c(1-\frac{2GM}{rc^2})\sqrt{1-\frac{r_{\min}^2(1-2GM/rc^2)}{r^2(1-2GM/r_{\min}c^2)}} \tag{44}$$

The time needed for radar echo to travel in gravitational field can be obtained by integrating the above two equations with respect to dt. As $GM/r_{\min}c^2<<1$, Equations (43) and (44) are the same, and the time delays are certainly the same.

4 The gravitational field produced by moving body

4.1 The gravitational field produced by moving body

We know that a rest charge produces only electrostatic field, but a moving charge produces electric field and magnetic field. Starting with Coulomb's law and special relativity, we can obtain the expression of magnetic field. From the similarity between electrostatic force and gravitational force, we may think that a moving body can produce gravitomagnetic field, too. We now start with the law of universal gravitation and special relativity to get the corresponding gravitomagnetic field expression.

Let’s first see how to obtain magnetic field from Coulomb’s law and special relativity [7]. Suppose a charge Q moves with a uniform velocity u relative to a rest inertial coordinate frame S, and another charge q moves with a velocity v with respect to S. We first write down the interaction of Q on q in the coordinate frame S' moving with charge Q, which is electrostatic force according to Coulomb’s law. Then according to the transformation formulas of force between inertial coordinate frames, we obtain the interaction of Q on q in the rest coordinate frame S. The corresponding gravitomagnetic force term will appear.

We wish to deal with the law of universal gravitation in the same manner. But we encounter difficulty for there are no synchronous clocks and unified rulers in the gravitational field. As shown in Figure 2, two particles move in gravitational field with their speed are u and v, respectively with respect to the gravitational source M. When gravitational field does not exist, according to the velocity addition formulas in special relativity, the relative speed for each particle is

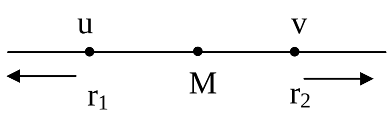

Figure 2. Two particles move relative to gravitational source

$$u' = \frac{u+v}{1-uv/c^2} \tag{45}$$

When gravitational field exists, considering the effects of time dilation and length contraction, we should first perform transformations, and then apply the velocity addition formula in special relativity. We make such transformations: $u = dr_1/dt_1 \rightarrow e^{\frac{2GM}{r_1c^2}} u$, $v = dr_2/dt_2 \rightarrow e^{\frac{2GM}{r_2c^2}} v$. Then applying Equation (45), we obtain

$$u' = \frac{e^{\frac{2GM}{r_1c^2}} u + e^{\frac{2GM}{r_2c^2}} v}{1-uv(e^{\frac{2GM}{r_1c^2}+\frac{2GM}{r_2c^2}})/c^2} \tag{46}$$

It can be seen that the circumstance becomes extraordinarily complicated when gravitational field exists. The reason that gravitational field is more complex than electromagnetic field is as follows: The existence of electromagnetic forces does not affect the speed of light, and we can establish synchronous clocks and unified rulers in space. While the existence of gravitational force affects the speed of light, and we cannot establish synchronous clocks and unified rulers in space. In order to apply Lorentz transformation to obtain gravitational field and gravitomagnetic field, we must first revise the standards of time and length in gravitational field. Of course, the results may be very complicated. We do not solve for the complicated transformation here, instead we solve for the approximate transformation formulas in weak gravitational field and nonrelativistic instance.

In weak gravitational field, $GM/rc^2 << 1$, the effects of time dilation and length contraction can be ignored. For nonrelativistic motion, the mass of body can be regarded as constant m_0 when making coordinate transformation. Thus we can make similar transformations as in electromagnetism. The force that a uniformly moving gravitational source M exerts on body m is

$$F = -\frac{GM_0 m_0}{\gamma^2 r^2 (1-\beta^2 \sin\theta)^{3/2}}[\mathbf{r}_1 + \frac{1}{c^2}\mathbf{v}\times(\mathbf{u}\times\mathbf{r}_1)] \tag{47}$$

where $\mathbf{u}$ is the velocity of the gravitational source M relative to the rest frame, $\mathbf{v}$ is the velocity of body m relative to the rest frame, $\mathbf{r}_1$ is the unit vector pointing from M to m, θ is the included angular between the vectors $\mathbf{u}$ and $\mathbf{r}_1$, $\beta = u/c$, $\gamma = 1/\sqrt{1-\beta^2}$. If we calculate the gravitational field and the gravitomagnetic field produced by a moving gravitational source, the expressions are

$$E_g = -\frac{GM_0}{\gamma^2 r^2 (1-\beta^2 \sin\theta)^{3/2}}\mathbf{r}_1 \tag{48}$$

$$B_g = -\frac{GM_0 u \sin\theta}{c^2 \gamma^2 r^2 (1-\beta^2 \sin\theta)^{3/2}}\boldsymbol{\varphi}_1 \tag{49}$$

where E_g and B_g are the gravitational field intensity and the gravitomagnetic field intensity, respectively, $\boldsymbol{\varphi}_1$ is the unit vector in the direction $\mathbf{u}\times\mathbf{r}_1$.

4.2 The predictions for GP-B test

"Gravity probe B" test is an example of weak gravitational field and nonrelativistic instance. The precession angular velocity of gyroscope in general relativity is [1]

$$\boldsymbol{\Omega} = -\frac{1}{2}\nabla\times\boldsymbol{\zeta} - \frac{3}{2}\frac{\mathbf{v}\times\nabla\phi}{c^2} \tag{50}$$

The first term on the right-hand side of the equation is frame-dragging effect, and the second term geodetic effect, where

$$\phi = -\frac{GM_\oplus}{r} \qquad \boldsymbol{\zeta} = \frac{2G}{r^3 c^2}(\mathbf{x}\times\mathbf{J}_\oplus) \tag{51}$$

Now we analyze GP-B test based on weak gravitational field and nonrelativistic hypothesis. Let's first see the Thomas precession in special relativity [8]. Suppose a charged particle rotates with respect to a laboratory inertial frame. The charged particle's rest frame of coordinate is defined as a co-moving sequence of inertial frames whose successive origins move at each instant with the velocity of the charged particle. The total time rate of the spin with respect to the laboratory inertial frame, or more generally, any vector $\mathbf{G}$ is given by the well-known result

$$\left(\frac{d\mathbf{G}}{dt}\right)_{notrot} = \left(\frac{d\mathbf{G}}{dt}\right)_{rest\ \ frame} + \boldsymbol{\omega}_T \times \mathbf{G} \tag{52}$$

where $\boldsymbol{\omega}_T$ is the angular velocity of rotation found by Thomas, whose expression is

$$\boldsymbol{\omega}_T = \frac{\gamma^2}{\gamma+1}\frac{\mathbf{a}\times\mathbf{v}}{c^2} \approx \frac{1}{2}\frac{\mathbf{a}\times\mathbf{v}}{c^2} \tag{53}$$

Suppose a charged particle moves with the speed of $\mathbf{v}$ in the external field of $\mathbf{E}$ and $\mathbf{B}$. In the charged particle's rest coordinate frame, the motion equation of the spin is

$$\left(\frac{d\mathbf{J}}{dt}\right)_{rest\ \ frame} = \boldsymbol{\mu}\times\mathbf{B}' \tag{54}$$

where $\boldsymbol{\mu}$ is the magnetic moment of the charged particle, and $\mathbf{B}'$ is the magnetic induction intensity in the charged particle's rest coordinate frame, which can be written

$$\mathbf{B}' = \gamma(\mathbf{B} - \frac{\mathbf{v}}{c^2}\times\mathbf{E}) \approx (\mathbf{B} - \frac{\mathbf{v}}{c^2}\times\mathbf{E}) \tag{55}$$

We know that the classical relation between magnetic moment $\boldsymbol{\mu}$ and angular momentum $\mathbf{J}$ of charged particle is [4]

$$\boldsymbol{\mu} = \frac{q}{2m}\mathbf{J} \tag{56}$$

Thus the motion equation of the spin of charged particle with respect to the laboratory inertial frame is

$$\frac{d\mathbf{J}}{dt} = \boldsymbol{\mu}\times\mathbf{B}' + \boldsymbol{\omega}_T\times\mathbf{J} = [\frac{q}{2m}(\frac{\mathbf{v}}{c^2}\times\mathbf{E} - \mathbf{B}) + \boldsymbol{\omega}_T]\times\mathbf{J} \tag{57}$$

Based on the similarity between gravitational force and electromagnetic force, we replace charge q with mass m, $\mathbf{E}$ and $\mathbf{B}$ with $\mathbf{E}_g$ and $\mathbf{B}_g$, respectively. Thus we obtain the motion equation of gyroscope's spin as

$$\frac{d\mathbf{J}}{dt} = [\frac{1}{2}(\frac{\mathbf{v}}{c^2}\times\mathbf{E}_g - \mathbf{B}_g) + \boldsymbol{\omega}_T]\times\mathbf{J} = [\frac{1}{2}(\frac{\mathbf{v}}{c^2}\times\mathbf{a} - \mathbf{B}_g) + \frac{\mathbf{a}\times\mathbf{v}}{2c^2}]\times\mathbf{J} = -\frac{1}{2}\mathbf{B}_g\times\mathbf{J} \tag{58}$$

It can be seen that the Thomas precession term is compensated with the term arising from the gravitational field intensity $\mathbf{E}_g$, and the geodetic precession is right zero. It may be a coincidence at first glance. In fact this is guaranteed by the principle of equivalence. We know that as the gyroscope falls freely in the gravitational field, the gravitational force on the gyroscope is balanced against its acceleration, and the gyroscope is a local inertial system. So the spin axis of the gyroscope remains unchanged with respect to the inertial coordinate frame at infinity. In fact, besides the centripetal attractive force, the gyroscope experiences gravitomagnetic force in the transverse direction. The principle of equivalence ensures that the free-falling gyroscope is shielded from not only the gravitational force but also the gravitomagnetic force. The change of the spin axis originates from the external gravitomagnetic moment on the spinning gyroscope, and the precession angular velocity is $-\frac{1}{2}\mathbf{B}_g$.

If this were not the case, the Thomas precession arising from the motion of the gyroscope round the Sun with the Earth could not be neglected. We know that the Sun is a precise inertial system, and it is at rest relative to the distant quasar(more generally, the distant quasar is at rest relative to the center of the Milky Way). Thus the motion of the spin with respect to the distant star equals to the motion relative to the Sun. We calculate and find it to be 6.3 milliarcseconds per year. Because the included angular between the equatorial plane and the orbital plane of the Earth is 23.5°, only part of $6.3\times\cos 23.5^\circ = 5.8$ milliarcseconds per year can lead to the precession of the gyroscope with the direction perpendicular to equatorial plane, i.e., in the direction of the frame-dragging effect. The other part is parallel to the spin axis of the gyroscope and cannot lead to precession. The principle of equivalence ensures that we can ignore all the external forces acted on the gyroscope and only consider the influence of external moment on the spin of the gyroscope.

In order to compute the gravitomagnetic field intensity $\mathbf{B}_g$ produced by the spin of the Earth, we introduce gravitomagnetic vector potential $\mathbf{A}_g$, which is similar to the magnetic vector potential in electrodynamics with the difference of replacement of current density with momentum density, and we also have $\mathbf{B}_g = \nabla\times\mathbf{A}_g$.

$$\mathbf{A}_g(\mathbf{x},t) = -\frac{G}{c^2}\int\frac{\rho\mathbf{v}(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|}d^3x' \tag{59}$$

Thus the precession angular velocity of gyroscope in external gravitomagnetic field is

$$\omega = -\frac{1}{2}\mathbf{B}_g = -\frac{1}{2}\nabla\times\mathbf{A}_g \tag{60}$$

While in general relativity, vector $\boldsymbol{\zeta}$ is introduced with the meaning similar to $\mathbf{A}_g$. But it is four times of $\mathbf{A}_g$ in magnitude [1].

$$\boldsymbol{\zeta}(\mathbf{x},t) = -\frac{4G}{c^2}\int\frac{\overset{1}{T}{}^{i0}(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|}d^3x' \tag{61}$$

The field of $\boldsymbol{\zeta}$ is given in Equation (51) for the Earth. Thus it can be seen that the frame-dragging effect in general relativity is four times larger than ours. The frame-dragging precession is 41 milliarcseconds per year in general relativity, so our result is $41/4\approx 10$ milliarcseconds per year.

We summarize our predictions for GP-B test as follows: The geodetic precession is zero and the frame-dragging precession is about 10 milliarcseconds per year.

Gravity Probe B spacecraft was launched on April 20,2004 [9], it will helps to make correct judgment, and we will wait and see the result.

4.3 Gravitational radiation

It is well known that as an electrical charge makes accelerated motion, it produces electromagnetic radiation. Likewise, we expect that a body will produce gravitational radiation as it makes accelerated motion. For an isolated charge system, the strongest radiation is electric dipole moment radiation. The definition of dipole moment is

$$\mathbf{d}_e = \sum_i e_i \mathbf{r}_i \tag{62}$$

where e_i and $\mathbf{r}_i$ are the charge and the position vector of particle i, respectively. The radiant intensity of dipole moment is proportional to $\ddot{\mathbf{d}}_e$. If we replace e_i in above equation with m_i, we obtain the definition of mass dipole moment of an isolated system.

$$\mathbf{d}_m = \sum_i m_i \mathbf{r}_i \tag{63}$$

whose first order derivative is the total momentum of the system.

$$\dot{\mathbf{d}}_m = \sum_i m_i \dot{\mathbf{r}}_i \tag{64}$$

As the total momentum of an isolated system is conservative, we have $\ddot{\mathbf{d}}_m = \dot{\mathbf{p}} = 0$. Thus mass dipole moment radiation cannot exist in gravity physics.

In electromagnetics, the second strongest radiations are magnetic dipole moment radiation and electric quadrupole moment radiation. The radiant intensity of magnetic dipole moment is determined by its second order derivative. The magnetic dipole moment can be written

$$\boldsymbol{\mu} = \frac{1}{2}\sum_i \mathbf{r} \times (e_i \mathbf{v}_i) \tag{65}$$

Replacing e_i in the above equation with m_i, we obtain the gravitomagnetic dipole moment

$$\boldsymbol{\mu}_g = \frac{1}{2}\sum_i \mathbf{r} \times (m_i \mathbf{v}_i) \tag{66}$$

which is just half of the angular momentum of the system. As the angular momentum of an isolated system is conservative, there does not exist gravitomagnetic dipole moment radiation.

The gravitational radiation similar to electric quadrupole moment radiation does exist. For an isolated system, the main gravitational radiation is mass quadrupole moment radiation. The definition of mass quadrupole moment is

$$\mathbf{D}_{\alpha\beta} = \int \rho(3x^{\alpha}x^{\beta} - \delta_{\alpha}^{\beta}x^{\gamma}x^{\gamma})dV \tag{67}$$

The total radiant power is

$$\frac{dE}{dt} == -\frac{G}{45c^5}(\dddot{\mathbf{D}}_{\alpha\beta})^2 \tag{68}$$

The above simple discussions of gravitational radiation can refer to [6], more detailed discussions may see [1],[3].

5 Black hole and quasar

From Schwarzschild solution of Einstein's gravitational field equation, one finds gravitational radius $r = 2GM/c^2$, which raises the problems of black hole and singularity. But in our theory, there are no such odd things. We know that the energy source of quasars is generally believed to be due to the accretion of black holes. If black hole does not exist, where does the huge energy of quasar come from?

In Ref [1] a type of suppermassive star is discussed, whose equilibrium is maintained by radiation pressure instead of matter pressure and whose mass is given by

$$M = 18M_S(\frac{m_N}{\overline{m}})^2\beta^{-2} \tag{69}$$

where M_S is the mass of the Sun, β is the ratio of matter pressure to radiation pressure. For the ionized hydrogen with the temperature between 10^5 K and 10^{10} K, $\overline{m}$ is the average mass of electron and proton. Under this circumstance, if radiation pressure is 10 times of matter pressure, i.e., β=0.1, we find $M = 7200M_S$. According to Eddington's mass-luminosity relation, the luminosity is $7200^{3.5} = 3.16\times10^{13}$ times brighter than that of the Sun and has reached the luminosity of common quasars. So quasars are most probable to be supermassive stars. The formations of supermassive stars require plenty of hydrogen gases, which are only possible at the earlier stage of the universe. We now see distant quasars only because the limited propagation speed of the light.

6 Conclusion

Our theory agrees with general relativity in the effects of time dilation and length contraction in gravitational field. As for the gravitational field equation, ours is based on the law of universal gravitation and the modified Lorentz covariant forms. While in general relativity, it is Einstein's gravitational field equation. For such difference exists, the predictions for GP-B test are different. We expect it will give a decisive judgment in the near future.

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